LABORATORY MANUAL

Higher Secondary Stage

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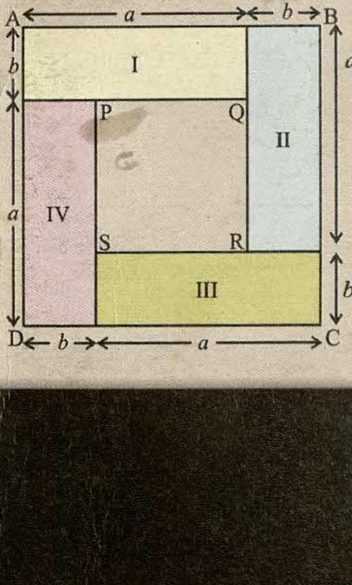

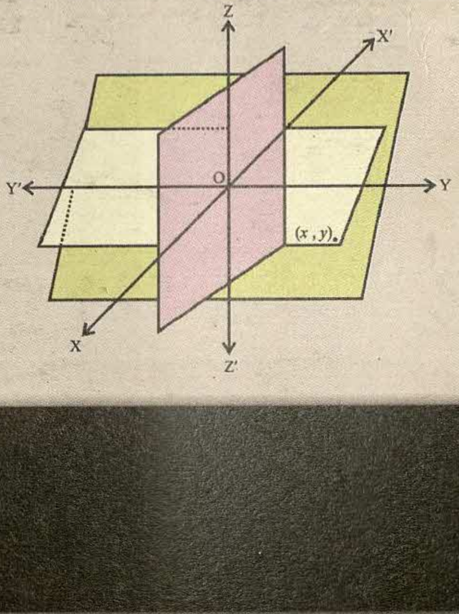

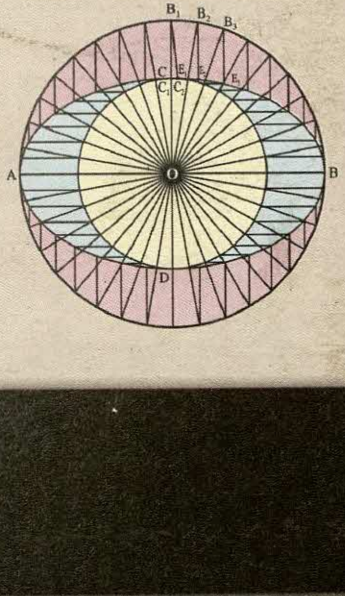

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Laboratory Manual

MATHEMATICS

Higher Secondary Stage

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NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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Foreword

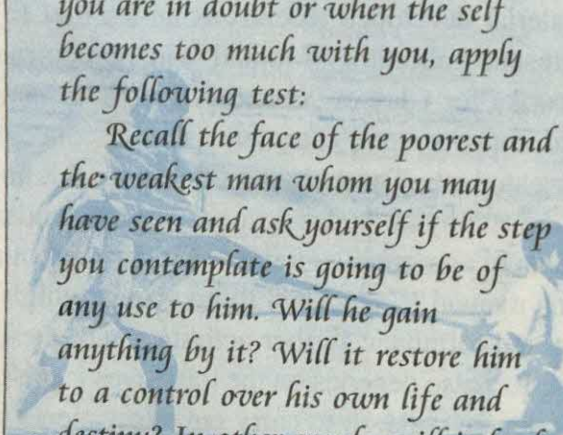
The National Council of Educational Research and Training (NCERT) is the apex body concerning all aspects of school education. It has recently developed textual material in Mathematics for the higher secondary stage which is based on the National Curriculum Framework (NCF) 2005. NCF-2005 recommends that children's experience in school education must be linked to the life outside school so that learning experience is joyful and fills the gap between the experience at home and in community. It recommends to diffuse the sharp boundaries between different subjects and discourages rote learning. The syllabi and the textual material developed recently is an attempt to implement this basic idea. The present Laboratory Manual will be complementary to the Mathematics textbooks for Classes XI and XII. It is in continuation to the NCERT's efforts to improve upon comprehension of concepts and practical/ activity skills among students. The purpose of this manual is not only to convey the approach and philosophy of the laboratory course to students and teachers but also to provide them appropriate guidance for carrying out experiments in the laboratory. This manual is supposed to encourage children to reflect on their own learning and to pursue further activities and questions. Of course the success of this effort also depends on the initiatives taken by the principals and teachers to encourage children to carry out experiments in the laboratory and to develop their own thinking and nurture creativity.

The methods adopted for performing the practicals/activities and their evaluation will determine how effective this practical/activity book will prove to make the children's life at school a happy experience, rather than a source of stress and boredom. This laboratory manual attempts to provide space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience. It is hoped that the material provided in this manual will help students in carrying out laboratory work effectively and will encourage teachers to introduce some open-ended experiments at the school level.

YASH PAL
Chairman
National Steering Committee
National Council of Educational Research and Training

New Delhi
21 May 2008

Gandhiji's Talisman



I will give you a talisman. Whenever you are in doubt or when the self becomes too much with you, apply the following test:

Recall the face of the poorest and the weakest man whom you may have seen and ask yourself if the step you contemplate is going to be of any use to him. Will he gain anything by it? Will it restore him to a control over his own life and destiny? In other words, will it lead to Swaraj for the hungry and spiritually starving millions?

Then you will find your doubts and your self melting away.

mkGandhi

Preface

The Laboratory Manual in Mathematics for Classes XI and XII is in continuation of our efforts in implementing the recommendations of National Curriculum Framework 2005 (NCF-2005). This manual is complementary to the Classes XI and XII Mathematics textbooks and aims at enhancing children's comprehension of scientific concepts as also acquiring basic experimental skills. In the learning of mathematics emphasis is on the enquiry approach and hands-on experience instead of lecture method alone. The recommendations of NCF-2005 on Teaching of Mathematics encourage experimental work and introduction of carefully designed experiments/activities. Schools may also be given a choice to select experiments according to the available infrastructure, cultural and environmental resources. At the higher secondary stage experimental work, often involving quantitative measurement as a tool to verify theoretical principles should form an integral part of the curriculum. This manual covers selected topics in the broad themes of Materials. It is aimed at motivating the reader to design an experiment, to make observations methodically and to draw logical conclusions. The experiments/activities are designed to expose the learners to basic tools and techniques of mathematical investigations.

Based on the mathematics curriculum up to classes XI and XII, sixty activities are given in this manual. All activities conform to a general format that includes – aim, theory, materials required, procedure, observations, results and discussion. Several experiments/activities also include 'a note' that suggests viable alternatives and clarifies certain anticipated difficulties while performing the experiment/activity. Further, applications are also quoted at several places to relate the concepts to different situations. Some experiments/activities have been left open-ended for teachers to innovate, modify and improve. Teachers may adapt or adopt these experiments/ activities for facilitating their teaching-learning processes.

It is a pleasure to express my thanks and gratitude to all those who have been involved at all stages during the development of this manual. I acknowledge the efforts of V. P. Singh, Coordinator of this programme and members of the team who contributed to the development and finalisation of the manual.

I express my gratitude to Professor G. Ravindra, *Joint Director*, NCERT for his administrative support and keen interest in the development of this manual. I am also grateful to the participating teachers and subject experts in the review workshop for their comments and suggestions which have helped in the refinement of this manual. We warmly welcome comments and suggestions from our readers for further improvement of this manual.

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Contents

Foreword iii

Preface v

The Purpose of the Mathematics Laboratory 1

Role of Mathematics Laboratory in Teaching-Learning 3

Management and Maintenance of Laboratory 4

Equipment for Mathematics Laboratory at the Higher Secondary Stage 7

Activities for Class XI 9-98

Activity 1 : To find the number of subsets of a given set and verify that if a set has *n* number of elements, then the total number of subsets is 2^n. 11

Activity 2 : To verify that for two sets A and B, $n(A \times B) = pq$ and the total number of relations from A to B is 2^{pq}, where $n(A) = p$ and $n(B) = q$. 13

Activity 3 : To represent set theoretic operations using Venn diagrams. 16

Activity 4 : To verify distributive law for three given non-empty sets A, B and C, that is, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ 20

Activity 5 : To identify a relation and a function. 22

Activity 6 : To distinguish between a Relation and a Function. 24

Activity 7 : To verify the relation between the degree measure and the radian measure of an angle 27

Activity 8 : To find the values of sine and cosine functions in second, third and fourth quadrants using their given values in first quadrant. 30

Activity 9 : To prepare a model to illustrate the values of sine function and cosine function for different angles which are multiples of $\frac{\pi}{2}$ and π. 35

Activity 10 : To plot the graphs of sin x, sin $2x$, 2 sin$x$ and sin $\frac{x}{2}$, using same coordinate axes. 38

Activity 11 : To inerpret geometrically the meaning of $i=\sqrt{-1}$ and its integral powers. 41

Activity 12 : To obtain a quadratic function with the help of linear functions graphically. 43

Activity 13 : To verify that the graph of a given inequality, say $5x + 4y - 40 < 0$, of the form $ax + by + c < 0$, $a, b > 0, c < 0$ represents only one of the two half planes. 45

Activity 14 : To find the number of ways in which three cards can be selected from given five cards. 48

Activity 15 : To construct a Pascal's triangle and to write binomial expansion for a given positive integral exponent. 50

Activity 16 : To obtain formula for the sum of squares of first n-natural numbers. 52

Activity 17 : An alternative approach to obtain formula for the sum of squares of first n natural numbers. 55

Activity 18 : To demonstrate that the Arithmetic mean of two different positive numbers is always greater than the Geometric mean. 58

Activity 19 : To establish the formula for the sum of the cubes of the first n natural numbers. 60

ACTIVITY 20 : To verify that the equation of a line passing through the point of intersection of two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is of the form $(a_1x + b_1y + c_1) + \lambda \ (a_2x + b_2y + c_2) = 0$. 63
ACTIVITY 21 : To construct different types of conic sections. 66
ACTIVITY 22 : To construct a parabola. 69
ACTIVITY 23 : An alternative method of constructing a parabola. 72
ACTIVITY 24 : To construct an ellipse using a rectangle. 75
ACTIVITY 25 : To construct an ellipse with given major and minor axes. 77
ACTIVITY 26 : To construct an ellipse when two fixed points are given. 79
ACTIVITY 27 : To explain the concept of octants by three mutually perpendicular planes in space. 81
ACTIVITY 28 : To find analytically $\lim\limits_{x \to c} f(x) = \dfrac{x^2 - c^2}{x - c}$ 84
ACTIVITY 29 : Verification of the geometrical significance of derivative. 86
ACTIVITY 30 : To obtain truth values of compound statements of the type $p \vee q$ by using switch connections in parallel. 90
ACTIVITY 31 : To obtain truth values of compound statements of the type $p \wedge q$ by using switch connections in series. 92
ACTIVITY 32 : To write the sample space, when a die is rolled once, twice -------- 94
ACTIVITY 33 : To write the sample space, when a coin is tossed once, two times, three times, four times 96

Activities for Class XII 99-175

Activity 1 : To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \perp m\}$ is symmetric but neither reflexive nor transitive. 101

Activity 2 : To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation. 103

Activity 3 : To demonstrate a function which is not one-one but is onto. 105

Activity 4 : To demonstrate a function which is one-one but not onto. 107

Activity 5 : To draw the graph of $\sin^{-1} x$, using the graph of $\sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$). 109

Activity 6 : To explore the principal value of the function $\sin^{-1} x$ using a unit circle. 112

Activity 7 : To sketch the graphs of a^x and $\log_a x$, $a > 0$, $a \neq 1$ and to examine that they are mirror images of each other. 116

Activity 8 : To establish a relationship between common logarithm (to the base 10) and natural logarithm (to the base e) of the number x. 119

Activity 9 : To find analytically the limit of a function $f(x)$ at $x = c$ and also to check the continuity of the function at that point. 122

Activity 10 : To verify that for a function f to be continuous at given point x_0, $\Delta y = |f(x_0 + \Delta x) - f(x_0)|$ is arbitrarily small provided. Δx is sufficiently small. 124

Activity 11 : To verify Rolle's Theorem. 127

Activity 12 : To verify Lagrange's Mean Value Theorem. 129

Activity 13 : To understand the concepts of decreasing and increasing functions. 132

ACTIVITY 14 : To understand the concepts of local maxima, local minima and point of inflection. 134

ACTIVITY 15 : To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph. 137

ACTIVITY 16 : To construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner. 140

ACTIVITY 17 : To find the time when the area of a rectangle of given dimensions become maximum, if the length is decreasing and the breadth is increasing at given rates. 143

ACTIVITY 18 : To verify that amongst all the rectangles of the same perimeter, the square has the maximum area. 147

ACTIVITY 19 : To evaluate the definite integral $\int_a^b \sqrt{(1-x^2)}\, dx$ as the limit of a sum and verify it by actual integration. 150

ACTIVITY 20 : To verify geometrically that $\vec{c}\times\left(\vec{a}+\vec{b}\right)=\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$ 153

ACTIVITY 21 : To verify that angle in a semi-circle is a right angle, using vector method. 157

ACTIVITY 22 : To locate the points to given coordinates in space, measure the distance between two points in space and then to verify the distance using distance formula. 160

ACTIVITY 23 : To demonstrate the equation of a plane in normal form. 163

ACTIVITY 24 : To verify that the angle between two planes is the same as the angle between their normals. 165

ACTIVITY 25 : To find the distance of given point (in space) from a plane (passing through three non-collinear points) by actual measurement and also analytically. 167

ACTIVITY 26 : To measure the shortest distance between two skew lines and verify it analytically. 171

ACTIVITY 27 : To explain the computaion of conditional probability of a given event A, when event B has already occurred, through an example of throwing a pair of dice. 174

Projects 176-182

PROJECT 1 : To minimise the cost of the food, meeting the dietary requirements of the staple food of the adoloscent students of your school. 177

PROJECT 2 : Estimation of the population of a particular region/country under the assumptions that there is no migration in or out of the existing population in a particular year. 178

PROJECT 3 : Finding the coordinates of different points identified in your classroom using the concepts of three dimensional geometry and also find the distances between the identified points. 180

PROJECT 4 : Formation of differential equation to explain the process of cooling of boiled water to a given room temperature. 181

LIST OF PROJECTS 183

SCHEME OF EVALUATION 185

The Purpose of the Mathematics Laboratory

National Policy on Education (1986) states "Mathematics should be visualised as a vehicle to train a child to think, analyse and articulate logically". National Curriculum Framework - 2005 brought out by NCERT states that the main goal of Mathematics education is mathematisation of child's thought process. These objectives can only be achieved if there is an opportunity of creating a scope of exploring, verifying and experimenting upon mathematical results by students themselves. Thus, there is need of adopting activity — oriented process rather than merely concentrating upon mastery of rules and formulae so as to do mathematical problems mechanically and pass out the examinations. There is need to provide the learners the scope for interaction, communication and representations of mathematical ideas by practising processes.

No doubt a laboratory is a place where scientific research and experiments are conducted for verification, exploration or discovery. Specifically, in mathematics the role of laboratory is helpful in understanding the mathematical concepts, formulae through activities. It is worth mentioning that pattern is central theme in mathematics which we need to develop practically to get insight into the mathematical concepts/theorems/formulae. Mathematics laboratory should not be solely a store house of teaching aids but in turn emphasis has to be laid on organising activities by students/teachers to rediscover the truth underlying the mathematical concepts. However, there may be a few interesting readymade geometrical and other models to motivate students. Moreover these models should be manipulative and dynamic.

A mathematics laboratory can foster mathematical awareness, skill building, positive attitude and learning by doing experiments in various topics of mathematics such as Algebra, Geometry, Mensuration, Trigonometry, Calculus, Coordinate Geometry, etc. It is the place where students can learn certain concepts using concrete objects and verify many mathematical facts and properties using models, measurements and other activities. It will also provide an opportunity to the students to do certain calculations using tables, calculators, etc., and also to listen or view certain audio-video cassettes relating to, remedial instructions, enrichment materials, etc. Mathematics laboratory will also provide an opportunity for the teacher to explain and demonstrate many mathematical concepts, facts and properties using concrete materials, models, charts, etc.

The teacher may also encourage students to prepare similar models and charts using materials like thermocol, cardboard, etc. in the laboratory. The laboratory will act as a forum for the teachers to discuss and deliberate on some important mathematical issues and problems of the day. It may also act as a place for teachers and the students to perform a number of mathematical celebrations and recreational activities.

Mathematics laboratory is expected to offer the following opportunities to learners:

- To discover the pattern for getting insight into the formulae
- To visualise algebraic and analytical results geometrically.
- To design practical demonstrations of mathematical results/formulae or the concepts.
- To encourage interactions amongst students and teachers through debate and discussions.
- To encourage students in recognising, extending, formulating patterns and enabling them to pose problems in the form of conjectures.
- To facilitate students in comprehending basic nature of mathematics from concrete to abstract.
- To provide opportunities to students of different ability groups in developing their skills of explaining and logical reasoning.
- To help students in constructing knowledge by themselves.
- To perform certain recreational activities in mathematics.
- To do certain projects under the proper guidance of the teacher.
- To explain visually some abstract concepts by using three dimensional models.
- To exhibit relatedness of mathematics with day to day life problems.

Role of Mathematics Laboratory in Teaching-Learning

Mathematics at Senior Secondary stage is a little more abstract as compared to the subject at the secondary stage. The mathematics laboratory at this stage can contribute in a big way to the learning of this subject.

Some of the ways are:

- Here the student will get an opportunity to understand the abstract ideas/ concepts through concrete objects and situations.
- The concepts of relations and functions can be easily understood by making working models and by making arrow diagrams using wires.
- Three dimensional concepts can only be conceived by three dimensional models in the laboratory, where as it is very difficult to understand these concepts on a black board.
- The concept of function and its inverse function, becomes very clear by drawing their graphs using mathematical instruments and using the concept of image about the line $y = x$, which can be done only in the laboratory.
- It provides greater scope for individual participation in the processes of learning and becoming autonomous learner.
- In the laboratory a student is encouraged to think, discuss with others and with the teacher. Thus, he can assimilate the concepts in a more effective manner.
- To the teacher also, mathematics laboratory enables to demonstrate and explain the abstract mathematical ideas, in a better way by using concrete objects, models etc.

Management and Maintenance of Laboratory

There is no second opinion that for effective teaching and learning 'Learning by doing' is of great importance as the experiences gained remains permanently affixed in the mind of the child. Exploring what mathematics is about and arriving at truth provides for pleasure of doing, understanding, developing positive attitude, and learning processes of mathematics and above all the great feeling of attachment with the teacher as facilitator. It is said 'a bad teacher teaches the truth but a good teacher teaches how to arrive at the truth.

A principle or a concept learnt as a conclusion through activities under the guidance of the teacher stands above all other methods of learning and the theory built upon it, can not be forgotten. On the contrary, a concept stated in the classroom and verified later on in the laboratory doesn't provide for any great experience nor make child's curiosity to know any good nor provides for any sense of achievement.

A laboratory is equipped with instruments, apparatus, equipments, models apart from facilities like water, electricity, etc. Non availability of a single material or facility out of these may hinder the performance of any experiment activity in the laboratory. Therefore, the laboratory must be well managed and well maintained.

A laboratory is managed and maintained by persons and the material required. Therefore, management and maintenance of a laboratory may be categorised as the personal management and maintenance and the material management and maintenance.

(A) PERSONAL MANAGEMENT AND MAINTENANCE

The persons who manage and maintain laboratories are generally called laboratory assistant and laboratory attendant. Collectively they are known as laboratory staff. Teaching staff also helps in managing and maintenance of the laboratory whenever and wherever it is required.

In personal management and maintenance following points are considered:

1. Cleanliness

A laboratory should always be neat and clean. When students perform experiment activities during the day, it certainly becomes dirty and things are scattered. So, it is the duty of the lab staff to clean the laboratory when the day's work is over and also place the things at their proper places if these are lying scattered.

2. **Checking and arranging materials for the day's work**

   Lab staff should know that what activities are going to be performed on a particular day. The material required for the day's activities must be arranged one day before.

   The materials and instruments should be arranged on tables before the class comes to perform an activity or the teacher brings the class for a demonstration.

3. The facilities like water, electricity, etc. must be checked and made available at the time of experiments.
4. It is better if a list of materials and equipments is pasted on the wall of the laboratory.
5. Many safety measures are required while working in laboratory. A list of such measures may be pasted on a wall of the laboratory.
6. While selecting the laboratory staff, the school authority must see that the persons should have their education with mathematics background.
7. A days training of 7 to 10 days may be arranged for the newly selected laboratory staff with the help of mathematics teachers of the school or some resource persons outside the school.
8. A first aid kit may be kept in the laboratory.

(B) MANAGEMENT AND MAINTENANCE OF MATERIALS

A laboratory requires a variety of materials to run it properly. The quantity of materials however depends upon the number of students in the school.

To manage and maintain materials for a laboratory following points must be considered:

1. A list of instruments, apparatus, activities and material may be prepared according to the experiments included in the syllabus of mathematics.
2. A group of mathematics teachers may visit the agencies or shops to check the quality of the materials and compare the rates. This will help to acquire the material of good quality at appropriate rates.

3. The materials required for the laboratory must be checked from time to time. If some materials or other consumable things are exhausted, orders may be placed for the same.
4. The instruments, equipments and apparatus should also be checked regularly by the laboratory staff. If any repair is required it should be done immediately. If any part is to be replaced, it should be ordered and replaced.
5. All the instruments, equipments, apparatus, etc. must be stored in the almirahs and cupboards in the laboratory or in a separate store room.

Equipment for Mathematics Laboratory at the Higher Secondary Stage

As the students will be involved in a lot of model making activities under the guidance of the teacher, the smooth running of the mathematics laboratory will depend upon the supply of oddments such as strings and threads, cellotape, white cardboard, hardboard, needles and pins, drawing pins, sandpaper, pliers, screw-drivers, rubber bands of different colours, gummed papers and labels, squared papers, plywood, scissors, saw, paint, soldering, solder wire, steel wire, cotton wool, tin and plastic sheets, glazed papers, etc. Besides these, some models, charts, slides, etc., made up of a good durable material should also be there for the teacher to demonstrate some mathematical concepts, facts and properties before the students. Different tables, ready reckner should also be there (in the laminated form) so that these can be used by the students for different purposes. Further, for performing activities such as measuring, drawing and calculating, consulting reference books, etc., there should be equipments like mathematical instruments, calculators, computers, books, journals mathematical dictionaries etc., in the laboratory.

In view of the above, following is the list of suggested instruments/models for the laboratory:

EQUIPMENT

Mathematical instrument set (Wooden Geometry Box for demonstration containing rulers, set-squares, divider, protractor and compasses), some geometry boxes, metre scales of 100 cm, 50 cm and 30 cm, measuring tape, diagonal scale, clinometer, calculators, computers including related software etc.

MODELS FOR DEMONSTRATION OF–

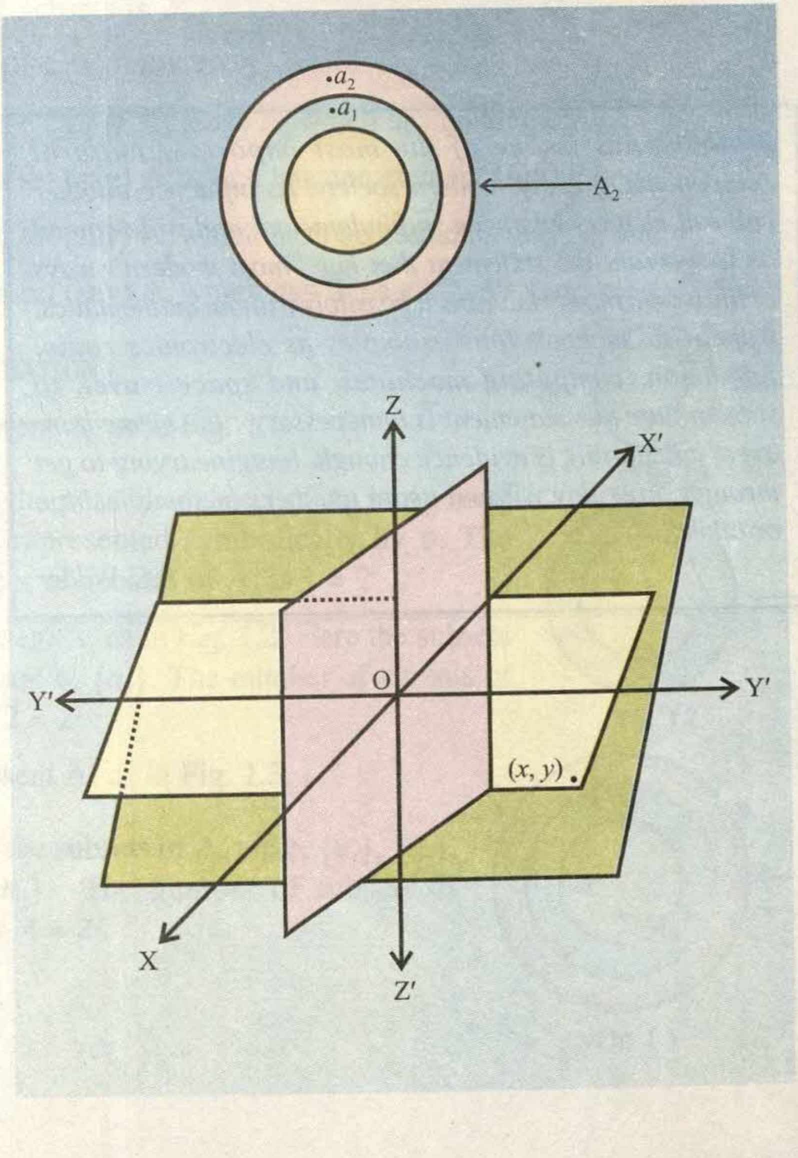
- Sets
- Relations and Functions
- Quadratic functions with the help of linear functions
- Sequence and series
- Pascal's triangle
- Arithmetic Progression

- Conic Sections
- Increasing, decreasing functions
- Maxima, minima, point of inflection
- Lagrange's minima, point of inflection
- Rolle's theorem
- Definite Integral as limit of sum
- Angle in semicircle using vectors
- Construction of parabola when distance between directrix and focus is given
- Construction of ellipse when major and minor axes are given
- Octants
- Shortest distance between two skew lines
- Geometrical interpretation of scalar and vector product
- Equation of a straight line passing through a fixed point and parallel to a given vector
- Equation to a plane
- Angle between two planes
- Bisection of the angles between two planes by a third plane
- Intersection of three planes
- Projection of the line segment
- Sample spaces
- Conditional Probability

STATIONERY AND ODDMENTS

Rubber-bands of different colours, Marbles of different colours, a pack of playing cards, graph paper/ squared paper, dotted paper, drawing pins, erasers, pencils, sketch pens, cellotapes, threads of different colours, glazed papers, kite papers, tracing papers, adhesive, pins, scissors and cutters, hammers, saw, thermocol sheets, sand paper, nails and screws of different sizes, screw drivers, drill machine with bit set, and pliers.

Activities for
Class XI



Mathematics is one of the most important cultural components of every modern society. Its influence another cultural element has been so fundamental and wide-spread as to warrant the statement that her "most modern" ways of life would hardly have been possilbly without mathematics. Appeal to such obvious examples as electronics radio, television, computing machines, and space travel, to substantiate this statement is unnecessary : the elementary art of calculating is evidence enough. Imagine trying to get through three day without using numbers in some fashion or other!

–R.L. Wilder

Activity 1

OBJECTIVE

To find the number of subsets of a given set and verify that if a set has *n* number of elements, then the total number of subsets is 2^n.

MATERIAL REQUIRED

Paper, different coloured pencils.

METHOD OF CONSTRUCTION

1. Take the empty set (say) A_0 which has no element.
2. Take a set (say) A_1 which has one element (say) a_1.
3. Take a set (say) A_2 which has two elements (say) a_1 and a_2.
4. Take a set (say) A_3 which has three elements (say) a_1, a_2 and a_3.

DEMONSTRATION

1. Represent A_0 as in Fig. 1.1

   Here the possible subsets of A_0 is A_0 itself only, represented symbolically by ϕ. The number of subsets of A_0 is $1 = 2^0$.

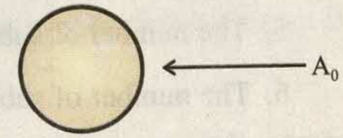


Fig. 1.1

2. Represent A_1 as in Fig. 1.2. Here the subsets of A_1 are ϕ, $\{a_1\}$. The number of subsets of A_1 is $2 = 2^1$

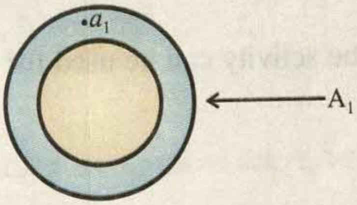


Fig. 1.2

3. Represent A_2 as in Fig. 1.3

   Here the subsets of A_2 are ϕ, $\{a_1\}$, $\{a_2\}$, $\{a_1, a_2\}$. The number of subsets of A_2 is $4 = 2^2$.

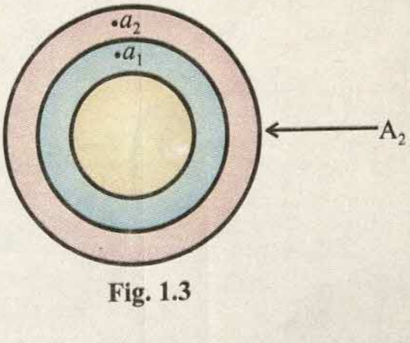


Fig. 1.3

4. Represent A_3 as in Fig. 1.4

   Here the subsets of A_3 are ϕ, $\{a_1\}$, $\{a_2\}$, $\{a_3)$,$\{a_1, a_2\}$, $\{a_2, a_3)$,$\{a_3, a_1)$ and $\{a_1, a_2, a_3\}$. The number of subsets of A_3 is $8 = 2^3$.

5. Continuing this way, the number of subsets of set A containing n elements $a_1, a_2, ..., a_n$ is 2^n.

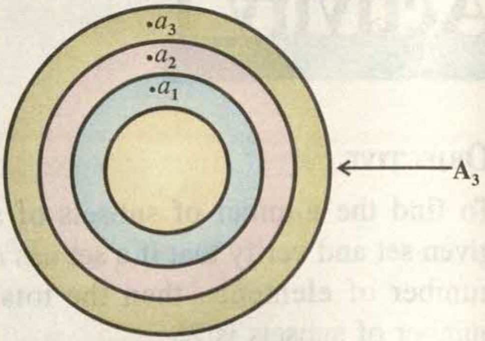


Fig. 1.4

OBSERVATION

1. The number of subsets of A_0 is ____________ $= 2^{\square}$
2. The number of subsets of A_1 is ____________ $= 2^{\square}$
3. The number of subsets of A_2 is ____________ $= 2^{\square}$
4. The number of subsets of A_3 is ____________ $= 2^{\square}$
5. The number of subsets of A_{10} is $= 2^{\square}$
6. The number of subsets of A_n is $= 2^{\square}$

APPLICATION

The activity can be used for calculating the number of subsets of a given set.

Activity 2

OBJECTIVE

To verify that for two sets A and B, $n(A\times B) = pq$ and the total number of relations from A to B is 2^{pq}, where $n(A) = p$ and $n(B) = q$.

MATERIAL REQUIRED

Paper, different coloured pencils.

METHOD OF CONSTRUCTION

1. Take a set A_1 which has one element (say) a_1, and take another set B_1, which has one element (say) b_1.
2. Take a set A_2 which has two elements (say) a_1 and a_2 and take another set B_3, which has three elements (say) b_1, b_2 and b_3.
3. Take a set A_3 which has three elements (say) a_1, a_2 and a_3, and take another set B_4, which has four elements (say) b_1, b_2, b_3 and b_4.

DEMONSTRATION

1. Represent all the possible correspondences of the elements of set A_1 to the elements of set B_1 visually as shown in Fig. 2.1.

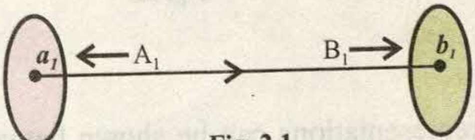


Fig. 2.1

2. Represent all the possible correspondences of the elements of set A_2 to the elements of set B_3 visually as shown in Fig. 2.2.

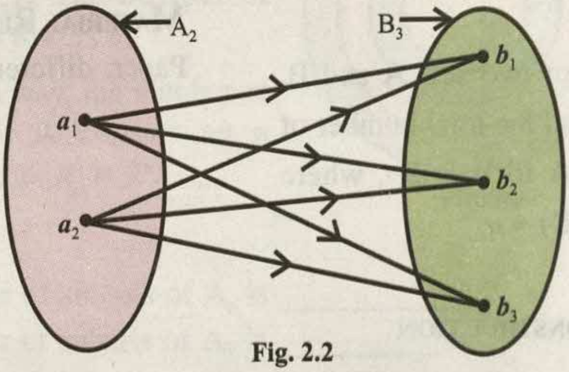


Fig. 2.2

3. Represent all the possible correspondences of the elements of set A_3 to the elements of set B_4 visually as shown in Fig. 2.3.

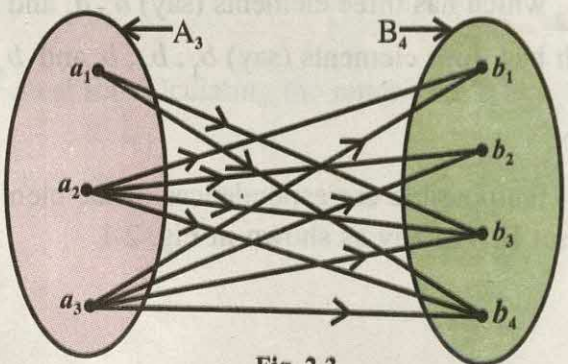


Fig. 2.3

4. Similar visual representations can be shown between the elements of any two given sets A and B.

OBSERVATION

1. The number of arrows, i.e., the number of elements in cartesian product $(A_1 \times B_1)$ of the sets A_1 and B_1 is _ × _ and the number of relations is $2^{\square}$.

2. The number of arrows, i.e., the number of elements in cartesian product $(A_2 \times B_3)$ of the sets A_2 and B_3 is _ × _ and number of relations is $2^{\square}$.

3. The number of arrows, i.e., the number of elements in cartesian product $(A_3 \times B_4)$ of the sets A_3 and B_4 is _ × _ and the number of relations is $2^{\square}$.

NOTE

The result can be verified by taking other sets $A_4, A_5, ..., A_p$, which have elements 4, 5,..., p, respectively, and the sets $B_5, B_6, ..., B_q$ which have elements 5, 6,..., q, respectively. More precisely we arrive at the conclusion that in case of given set A containing p elements and the set B containing q elements, the total number of relations from A to B is 2^{pq}, where $n(A \times B) = n(A)\, n(B) = pq$.

Activity 3

OBJECTIVE

To represent set theoretic operations using Venn diagrams.

MATERIAL REQUIRED

Hardboard, white thick sheets of paper, pencils, colours, scissors, adhesive.

METHOD OF CONSTRUCTION

1. Cut rectangular strips from a sheet of paper and paste them on a hardboard. Write the symbol U in the left/right top corner of each rectangle.
2. Draw circles A and B inside each of the rectangular strips and shade/colour different portions as shown in Fig. 3.1 to Fig. 3.10.

DEMONSTRATION

1. U denotes the universal set represented by the rectangle.
2. Circles A and B represent the subsets of the universal set U as shown in the figures 3.1 to 3.10.
3. A' denote the complement of the set A, and B' denote the complement of the set B as shown in the Fig. 3.3 and Fig. 3.4.
4. Coloured portion in Fig. 3.1. represents $A \cup B$.

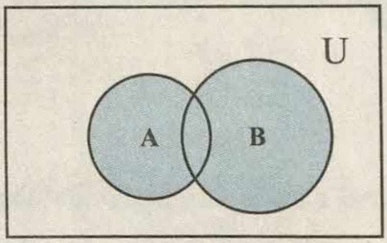


Fig. 3.1

5. Coloured portion in Fig. 3.2. represents $A \cap B$.

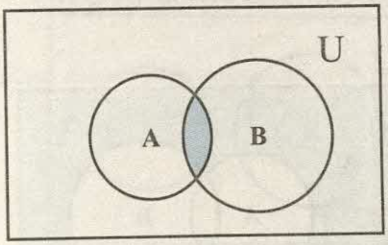


Fig. 3.2

6. Coloured portion in Fig. 3.3 represents A'

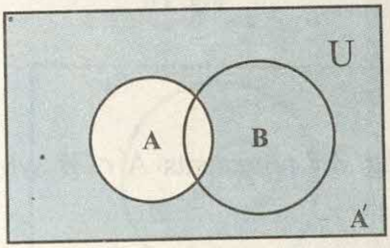


Fig. 3.3

7. Coloured portion in Fig. 3.4 represents B'

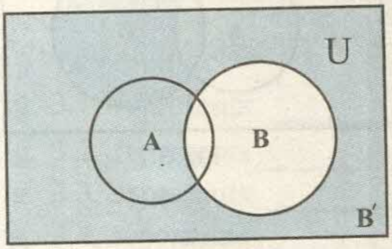


Fig. 3.4

8. Coloured portion in Fig. 3.5 represents $(A \cap B)'$

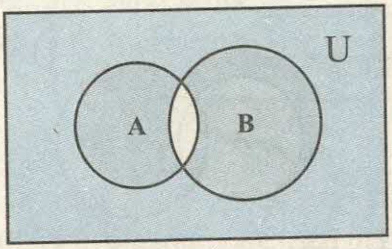


Fig. 3.5

9. Coloured portion in Fig. 3.6 represents $(A \cup B)'$

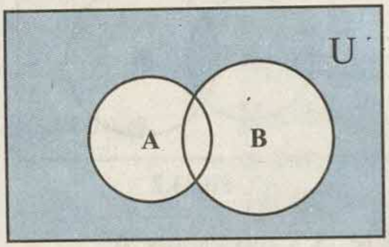


Fig. 3.6

10. Coloured portion in Fig. 3.7 represents $A' \cap B$ which is same as $B - A$.

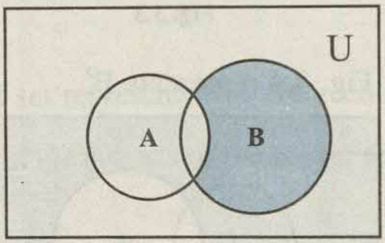


Fig. 3.7

11. Coloured portion in Fig. 3.8 represents $A' \cup B$.

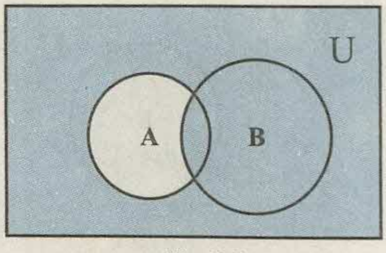


Fig. 3.8

12. Fig. 3.9 shows $A \cap B = \phi$

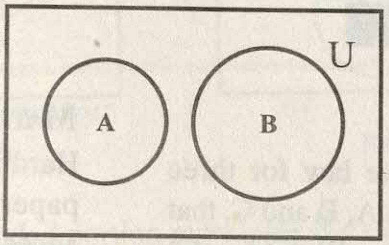


Fig. 3.9

13. Fig. 3.10 shows $A \subset B$

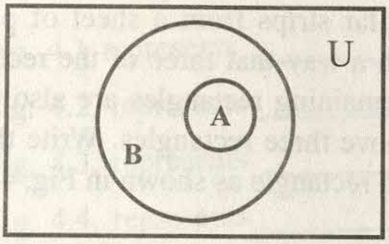


Fig. 3.10

OBSERVATION

1. Coloured portion in Fig. 3.1, represents ____________
2. Coloured portion in Fig. 3.2, represents ____________
3. Coloured portion in Fig. 3.3, represents ____________
4. Coloured portion in Fig. 3.4, represents ____________
5. Coloured portion in Fig. 3.5, represents ____________
6. Coloured portion in Fig. 3.6, represents ____________
7. Coloured portion in Fig. 3.7, represents ____________
8. Coloured portion in Fig. 3.8, represents ____________
9. Fig. 3.9, shows that $(A \cap B) =$ ____________.
10. Fig. 3.10, represents A ____________ B.

APPLICATION

Set theoretic representation of Venn diagrams are used in Logic and Mathematics.

Activity 4

OBJECTIVE

To verify distributive law for three given non-empty sets A, B and C, that is, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

MATERIAL REQUIRED

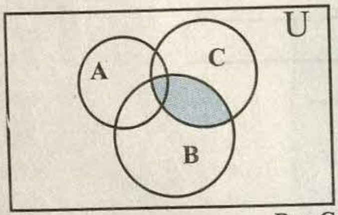
Hardboard, white thick sheets of paper, pencil, colours, scissors, adhesive.

METHOD OF CONSTRUCTION

1. Cut five rectangular strips from a sheet of paper and paste them on th hardboard in such a way that three of the rectangles are in horizontal lin and two of the remaining rectangles are also placed horizontally in a lin just below the above three rectangles. Write the symbol U in the left/righ top corner of each rectangle as shown in Fig. 4.1, Fig. 4.2, Fig. 4.3, Fig. 4. and Fig. 4.5.
2. Draw three circles and mark them as A, B and C in each of the five rectangle as shown in the figures.
3. Colour/shade the portions as shown in the figures.

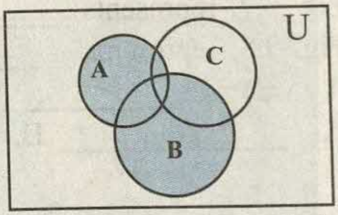
DEMONSTRATION

1. U denotes the universal set represented by the rectangle in each figure.
2. Circles A, B and C represent the subsets of the universal set U.



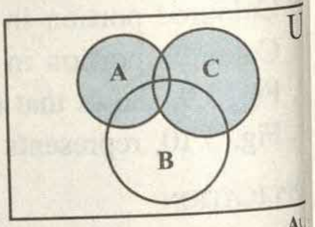
$B \cap C$

Fig. 4.1



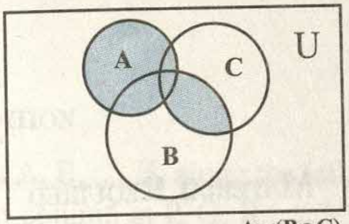
$A \cup B$

Fig. 4.2



$A \cup$

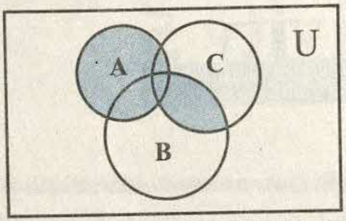
Fig. 4.3



=

A∪(B∩C)

Fig. 4.4



(A∪B)∩(A∪C)

Fig. 4.5

3. In Fig. 4.1, coloured/shaded portion represents B ∩ C, coloured portions in Fig. 4.2 represents A ∪ B, Fig. 4.3 represents A ∪ C, Fig. 4.4 represents A ∪ (B ∩ C) and coloured portion in Fig. 4.5 represents (A ∪ B) ∩ (A∪ C).

OBSERVATION

1. Coloured portion in Fig. 4.1 represents __________.
2. Coloured portion in Fig. 4.2, represents __________.
3. Coloured portion in Fig. 4.3, represents __________.
4. Coloured portion in Fig. 4.4, represents __________.
5. Coloured portion in Fig. 4.5, represents __________.
6. The common coloured portions in Fig. 4.4 and Fig. 4.5 are __________.
7. A∪(B∩C)= __________.

Thus, the distributive law is verified.

APPLICATION

Distributivity property of set operations is used in the simplification of problems involving set operations.

NOTE

In the same way, the other distributive law

A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

can also be verified.

6.07.2011

Activity 5

OBJECTIVE

To identify a relation and a function.

MATERIAL REQUIRED

Hardboard, battery, electric bulbs of two different colours, testing screws, tester, electrical wires and switches.

METHOD OF CONSTRUCTION

1. Take a piece of hardboard of suitable size and paste a white paper on it.
2. Drill eight holes on the left side of board in a column and mark them as A, B, C, D, E, F, G and H as shown in the Fig.5.
3. Drill seven holes on the right side of the board in a column and mark them as P, Q, R, S, T, U and V as shown in the Figure 5.
4. Fix bulbs of one colour in the holes A, B, C, D, E, F, G and H.
5. Fix bulbs of the other colour in the holes P, Q, R, S, T, U and V.

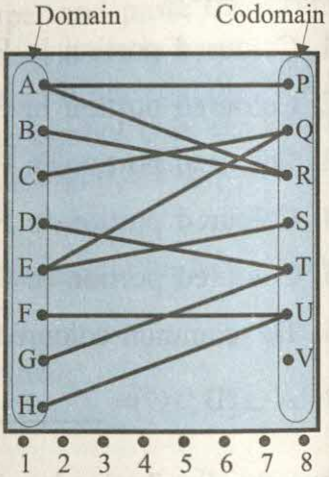


Fig. 5

6. Fix testing screws at the bottom of the board marked as 1, 2, 3, ..., 8.
7. Complete the electrical circuits in such a manner that a pair of corresponding bulbs, one from each column glow simultaneously.
8. These pairs of bulbs will give ordered pairs, which will constitute a relation which in turn may /may not be a function [see Fig. 5].

DEMONSTRATION

1. Bulbs at A, B, ..., H, along the left column represent domain and bulbs along the right column at P, Q, R, ..., V represent co-domain.
2. Using two or more testing screws out of given eight screws obtain different order pairs. In Fig.5, all the eight screws have been used to give different ordered pairs such as (A, P), (B, R), (C, Q) (A, R), (E, Q), etc.
3. By choosing different ordered pairs make different sets of ordered pairs.

OBSERVATION

1. In Fig.5, ordered pairs are ________.
2. These ordered pairs constitute a _______.
3. The ordered pairs (A, P), (B, R), (C, Q), (E, Q), (D, T), (G, T), (F, U), (H, U) constitute a relation which is also a _____.
4. The ordered pairs (B, R), (C, Q), (D, T), (E, S), (E, Q) constitute a _____ which is not a _____.

APPLICATION

The activity can be used to explain the concept of a relation or a function. It can also be used to explain the concept of one-one, onto functions.

Activity 6

OBJECTIVE

To distinguish between a Relation and a Function.

MATERIAL REQUIRED

Drawing board, coloured drawing sheets, scissors, adhesive, strings, nails etc.

METHOD OF CONSTRUCTION

1. Take a drawing board/a piece of plywood of convenient size and paste a coloured sheet on it.
2. Take a white drawing sheet and cut out a rectangular strip of size 6 cm × 4 cm and paste it on the left side of the drawing board (see Fig. 6.1)

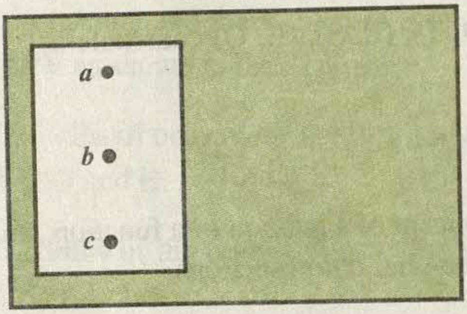


Fig. 6.1

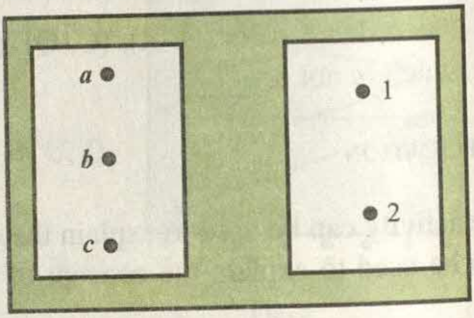


Fig. 6.2

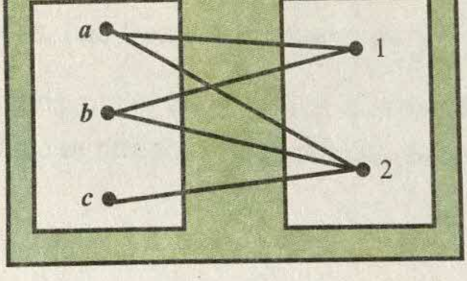


Fig. 6.3

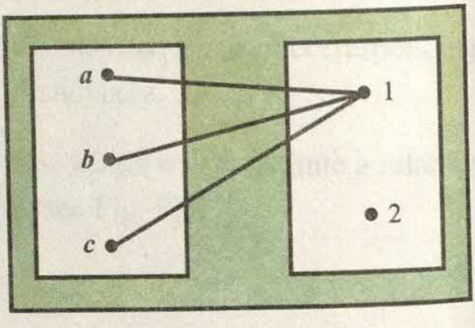


Fig. 6.4

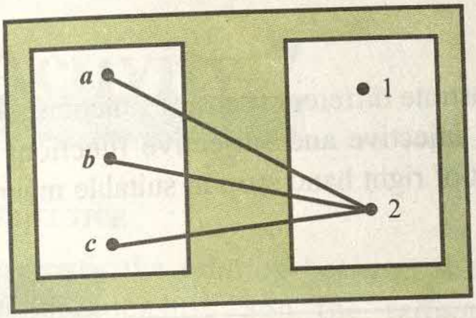


Fig. 6.5

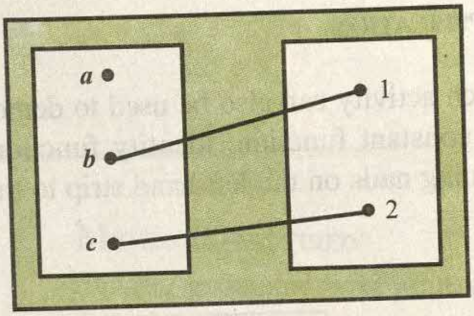


Fig. 6.6

3. Fix three nails on this strip and mark them as *a, b, c* (see Fig. 6.1).
4. Cut out another white rectangular strip of size 6 cm × 4 cm and paste it on the right hand side of the drawing board.
5. Fix two nails on the right side of this strip (see Fig. 6.2) and mark them as 1 and 2.

DEMONSTRATION

1. Join nails of the left hand strip to the nails on the right hand strip by strings in different ways. Some of such ways are shown in Fig. 6.3 to Fig. 6.6.
2. Joining nails in each figure constitute different ordered pairs representing elements of a relation.

OBSERVATION

1. In Fig. 6.3, ordered pairs are __________.
These ordered pairs constitute a __________ but not a __________.
2. In Fig. 6.4, ordered pairs are __________. These constitute a ________ as well as ________.
3. In Fig 6.5, ordered pairs are ________. These ordered pairs constitute a ________ as well as ________.
4. In Fig. 6.6, ordered pairs are ________. These ordered pairs do not represent ______ but represent ________.

APPLICATION

Such activity can also be used to demonstrate different types of functions such as constant function, identity function, injective and surjective functions by joining nails on the left hand strip to that of right hand strip in suitable manner.

NOTE

In the above activity nails have been joined in some different ways. The student may try to join them in other different ways to get more relations of different types. The number of nails can also be changed on both sides to represent different types of relations and functions.

Activity 7

OBJECTIVE

To verify the relation between the degree measure and the radian measure of an angle.

MATERIAL REQUIRED

Bangle, geometry box, protractor, thread, marker, cardboard, white paper.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw a circle using a bangle on the white paper.
3. Take a set square and place it in two different positions to find diameters PQ and RS of the circle as shown in the Fig.7.1 and 7.2

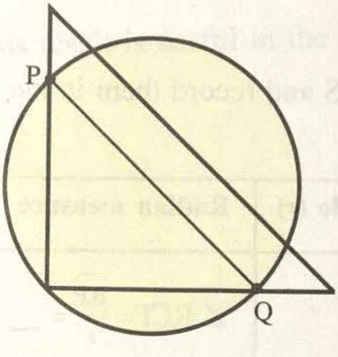


Fig. 7.1

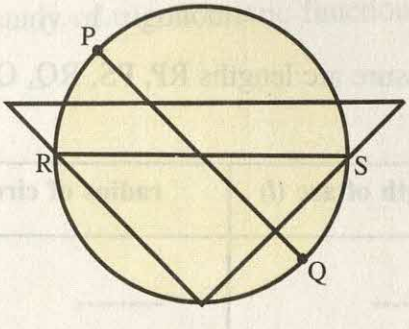


Fig. 7.2

4. Let PQ and RS intersect at C. The point C will be the centre of the circle (Fig. 7.3).
5. Clearly CP = CR = CS = CQ = radius.

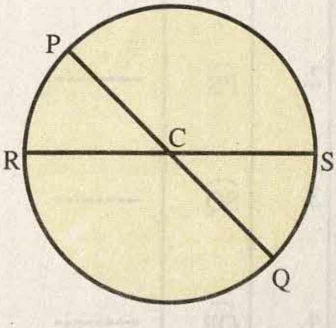


Fig. 7.3

DEMONSTRATION

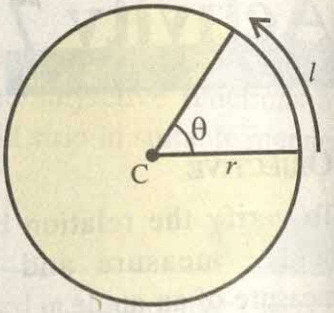


Fig. 7.4

1. Let the radius of the circle be r and l be an arc subtending an angle θ at the centre C, as shown in Fig. 7.4. $\theta = \frac{l}{r}$ radians.

2. If Degree measure of $\theta = \frac{l}{2\pi r} \times 360$ degrees

Then $\frac{l}{r}$ radians $= \frac{l}{2\pi r} \times 360$ degrees

or 1 radian $= \frac{180}{\pi}$ degrees = 57.27 degrees.

OBSERVATION

Using thread, measure arc lengths RP, PS, RQ, QS and record them in the table given below :

S.No	Arc	length of arc (l)	radius of circle (r)	Radian measure
1.	$\widehat{RP}$	--------	--------	$\angle RCP = \frac{\widehat{RP}}{r} =$ _
2.	$\widehat{PS}$	--------	--------	$\angle PCS = \frac{\widehat{PS}}{r} =$ _
3.	$\widehat{SQ}$	--------	--------	$\angle SCQ = \frac{\widehat{SQ}}{r} =$ _
4.	$\widehat{QR}$	--------	--------	$\angle QCR = \frac{\widehat{QR}}{r} =$ _

2. Using protractor, measure the angle in degrees and complete the table.

Angle	Degree measure	Radian Measure	Ratio = $\frac{\text{Degree measure}}{\text{Radian measure}}$
∠ RCP	--------	--------	--------
∠ PCS	--------	--------	--------
∠ QCS	--------	--------	--------
∠ QCR	--------	--------	--------

3. The value of one radian is equal to ________ degrees.

APPLICATION

This result is useful in the study of trigonometric functions.

Activity 8

OBJECTIVE

To find the values of sine and cosine functions in second, third and fourth quadrants using their given values in first quadrant.

MATERIAL REQUIRED

Cardboard, white chart paper, ruler, coloured pens, adhesive, steel wires and needle.

METHOD OF CONSTRUCTION

1. Take a cardboard of convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on chart paper.
3. Through the centre of the circle, draw two perpendicular lines X′OX and YOY′ representing x-axis and y-axis, respectively, as shown in Fig.8.1.

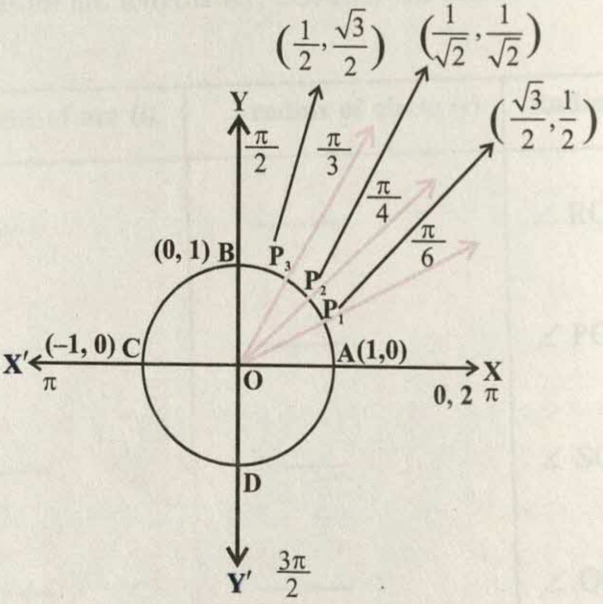

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
Y
$\frac{\pi}{2}$
$\frac{\pi}{3}$
$\frac{\pi}{4}$
$\frac{\pi}{6}$
(0, 1) B
P_3
P_2
P_1
X′
(–1, 0) C
π
O
A(1,0)
X
0, 2 π
D
Y′
$\frac{3\pi}{2}$

Fig. 8.1

4. Mark the points as A, B, C and D, where the circle cuts the *x*-axis and *y*-axis, respectively, as shown in Fig. 8.1.

5. Through O, draw angles P_1OX, P_2OX, and P_3OX of measures $\frac{\pi}{6}, \frac{\pi}{4}$ and $\frac{\pi}{3}$, respectively.

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle.

DEMONSTRATION

1. The coordinates of the point P_1 are $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ because its *x*-coordinate is $\cos\frac{\pi}{6}$ and *y*-coordinate is $\sin\frac{\pi}{6}$. The coordinates of the points P_2 and P_3 are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, respectively.

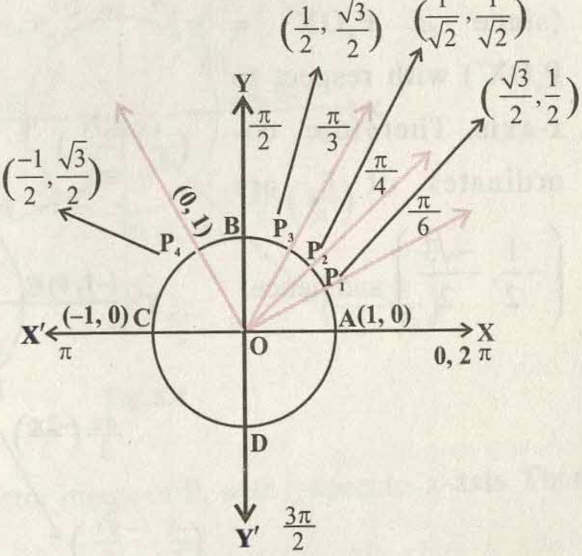

$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
Y
$\frac{\pi}{2}$
$\frac{\pi}{3}$
$\frac{\pi}{4}$
$\frac{\pi}{6}$
$(\frac{-1}{2}, \frac{\sqrt{3}}{2})$
(0, 1)
B
P_3
P_2
P_1
P_4
(–1, 0) C
A(1, 0)
X′
X
π
O
0, 2 π
D
Y′
$\frac{3\pi}{2}$

Fig. 8.2

2. To find the value of sine or cosine of some angle in the second quadrant (say) $\frac{2\pi}{3}$, rotate the needle in anti clockwise direction making an angle P_4OX of measure $\frac{2\pi}{3} = 120^\circ$ with the positive direction of *x*-axis.

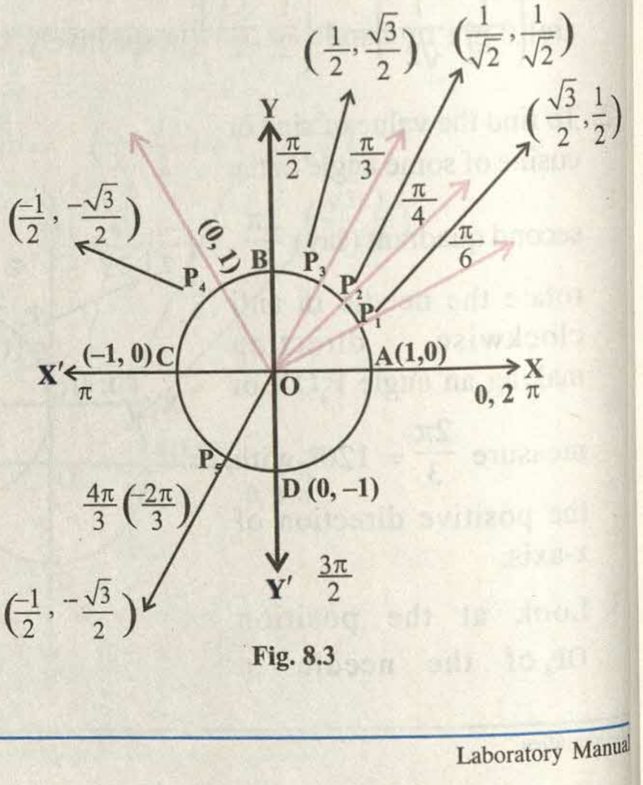
3. Look at the position OP_4 of the needle in

Fig.8.2. Since $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, OP_4 is the mirror image of OP_3 with respect to y-axis. Therefore, the coordinate of P_4 are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Thus

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{2\pi}{3} = -\frac{1}{2}$.

4. To find the value of sine or cosine of some angle say, $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$, i.e., $\frac{-2\pi}{3}$ (say) in the third quadrant, rotate the needle in anti clockwise direction making as an angle of $\frac{4\pi}{3}$ with the positive direction of x-axis.

5. Look at the new position OP_5 of the needle, which is shown in Fig. 8.3. Point P_5 is the mirror image of the point P_4 (since $\angle P_4OX' = P_5OX'$) with respect to x-axis. Therefore, co-ordinates of P_5 are $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ and hence

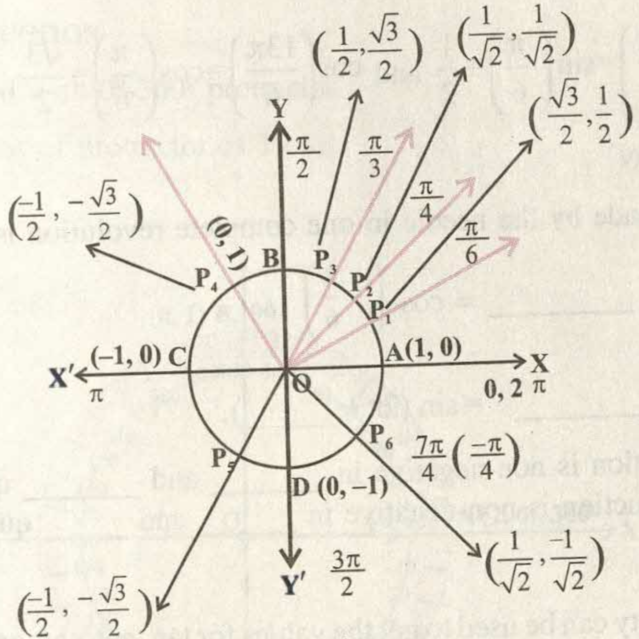


$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
Y
$\frac{\pi}{2}$
$\frac{\pi}{3}$
$\frac{\pi}{4}$
$\frac{\pi}{6}$
$\left(\frac{-1}{2}, -\frac{\sqrt{3}}{2}\right)$
(0, 1)
B
P_3
P_4
P_2
P_1
A(1,0)
(−1, 0) C
X′
π
O
X
0, 2 π
P_5
D (0, −1)
$\frac{4\pi}{3}\left(\frac{-2\pi}{3}\right)$
Y′
$\frac{3\pi}{2}$
$\left(\frac{-1}{2}, -\frac{\sqrt{3}}{2}\right)$

Fig. 8.3

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } \cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}.$$

6. To find the value of sine or cosine of some angle in the fourth quadrant, say $\frac{7\pi}{4}$, rotate the needle in anti clockwise direction making an angle of $\frac{7\pi}{4}$ with the positive direction of x-axis represented by OP_6, as shown in Fig. 8.4. Angle $\frac{7\pi}{4}$ in anti clockwise direction = Angle $-\frac{\pi}{4}$ in the clockwise direction.



$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
Y
$\frac{\pi}{2}$
$\frac{\pi}{3}$
$\frac{\pi}{4}$
$\frac{\pi}{6}$
$\left(\frac{-1}{2}, -\frac{\sqrt{3}}{2}\right)$
(0, 1)
B
P_3
P_4
P_2
P_1
(–1, 0) C
A(1, 0)
X′
X
π
O
0, 2 π
P_6
$\frac{7\pi}{4}\left(\frac{-\pi}{4}\right)$
P_5
D (0, –1)
$\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$
Y′
$\frac{3\pi}{2}$
$\left(\frac{-1}{2}, -\frac{\sqrt{3}}{2}\right)$

Fig. 8.4

From Fig. 8.4, P_6 is the mirror image of P_2 with respect to x-axis. Therefore, coordinates of P_6 are $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Thus $\sin\left(\frac{7\pi}{4}\right)=\sin\left(-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

and $\cos\left(\frac{7\pi}{4}\right)=\cos\left(-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

8. To find the value of sine or cosine of some angle, which is greater than one revolution, say $\frac{13\pi}{6}$, rotate the needle in anti clockwise direction since $\frac{13\pi}{6}=2\pi+\frac{\pi}{6}$, the needle will reach at the position OP_1. Therefore, $\sin\left(\frac{13\pi}{6}\right)=\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ and $\cos\left(\frac{13\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

OBSERVATION

1. Angle made by the needle in one complete revolution is _________.
2. $\cos\frac{\pi}{6}=$ _________ $=\cos\left(-\frac{\pi}{6}\right)$

   $\sin\frac{\pi}{6}=$ _________ $=\sin(2\pi+$ _______ $)$.
3. sine function is non-negative in _______ and _______ quadrants.
4. cosine function is non-negative in ______ and _____ quadrants.

APPLICATION

1. The activity can be used to get the values for tan, cot, sec, and cosec functions also.
2. From this activity students may learn that

   $\sin(-\theta)=-\sin\theta$ and $\cos(-\theta)=-\cos\theta$

   This activity can be applied to other trigonometric functions also.

Activity 9

OBJECTIVE

To prepare a model to illustrate the values of sine function and cosine function for different angles which are multiples of $\frac{\pi}{2}$ and π.

MATERIAL REQUIRED

A stand fitted with 0º-360º protractor and a circular plastic sheet fixed with handle which can be rotated at the centre of the protractor.

METHOD OF CONSTRUCTION

1. Take a stand fitted with 0º-360º protractor.
2. Consider the radius of protractor as 1 unit.

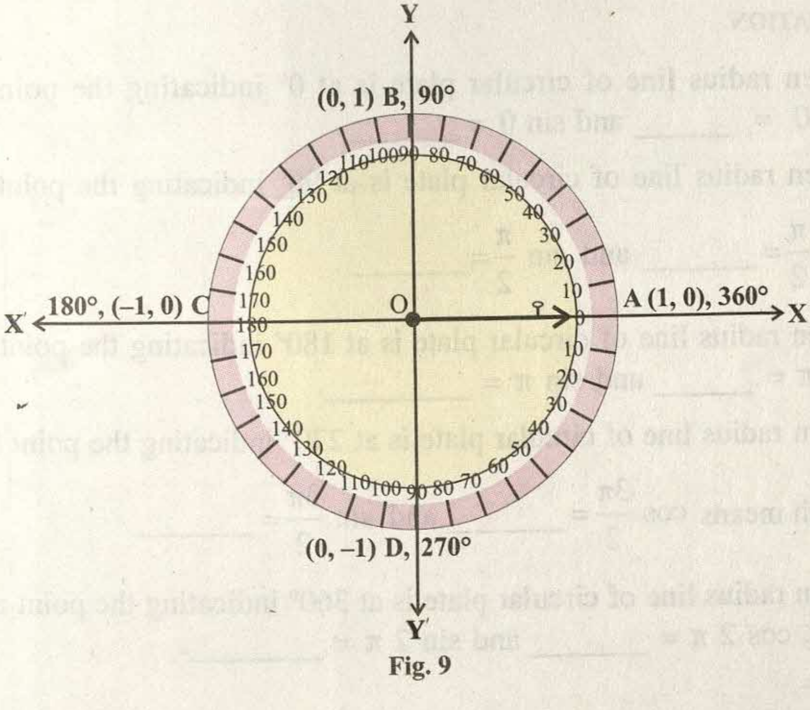


Fig. 9

3. Draw two lines, one joining 0°-180° line and another 90°-270° lin obviously perpendicular to each other.

4. Mark the ends of 0°-180° line as (1,0) at 0°, (–1, 0) at 180° and that 90° - 270° line as (0,1) at 90° and (0, –1) at 270°

5. Take a plastic circular plate and mark a line to indicate its radius and fix handle at the outer end of the radius.

6. Fix the plastic circular plate at the centre of the protractor.

DEMONSTRATION

1. Move the circular plate in anticlock wise direction to make different angl like $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ etc.

2. Read the values of sine and cosine function for these angles and the multiples from the perpendicular lines.

OBSERVATION

1. When radius line of circular plate is at 0° indicating the point A (1,0 $\cos 0 =$ ______ and $\sin 0 =$ ______.

2. When radius line of circular plate is at 90° indicating the point B (0, 1 $\cos \frac{\pi}{2} =$ ______ and $\sin \frac{\pi}{2} =$ ______.

3. When radius line of circular plate is at 180° indicating the point C (–1,0 $\cos \pi =$ ______ and $\sin \pi =$ ______.

4. When radius line of circular plate is at 270° indicating the point D (0, – which means $\cos \frac{3\pi}{2} =$ ______ and $\sin \frac{3\pi}{2} =$ ______

5. When radius line of circular plate is at 360° indicating the point again at (1,0), $\cos 2\pi =$ ______ and $\sin 2\pi =$ ______.

Now fill in the table :

Trigonometric function	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
$\sin\theta$	–	–	–	–	–	–	–	–	–
$\cos\theta$	–	–	–	–	–	–	–	–	–

APPLICATION

This activity can be used to determine the values of other trigonometric functions for angles being multiple of $\frac{\pi}{2}$ and π.

Activity 10

OBJECTIVE

To plot the graphs of sin x, sin $2x$, 2sin$x$ and sin$\frac{x}{2}$, using same coordinate axes.

MATERIAL REQUIRED

Plyboard, squared paper, adhesiv ruler, coloured pens, eraser.

METHOD OF CONSTRUCTION

1. Take a plywood of size 30 cm × 30 cm.
2. On the plywood, paste a thick graph paper of size 25 cm × 25 cm.
3. Draw two mutually perpendicular lines on the squared paper, and take them as coordinate axes.
4. Graduate the two axes as shown in the Fig. 10.
5. Prepare the table of ordered pairs for sin x, sin $2x$, 2sin x and sin $\frac{x}{2}$ fo different values of x shown in the table below:

T. ratios	**0°**	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{9\pi}{12}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
sin x	0	0.26	0.50	0.71	0.86	0.97	1.00	0.97	0.86	0.71	0.50	0.26	0
sin $2x$	0	0.50	0.86	1.00	0.86	0.50	0	–0.5	–0.86	–1.0	–0.86	–0.50	0
2 sin x	0	0.52	1.00	1.42	1.72	1.94	2.00	1.94	1.72	1.42	1.00	0.52	0
sin $\frac{x}{2}$	0	0.13	0.26	0.38	0.50	0.61	0.71	0.79	0.86	0.92	0.97	0.99	1.0

DEMONSTRATION

1. Plot the ordered pair $(x, \sin x)$, $(x, \sin 2x)$, $(x, \sin \frac{x}{2})$ and $(x, 2\sin x)$ on the same axes of coordinates, and join the plotted ordered pairs by free hand curves in different colours as shown in the Fig.10.

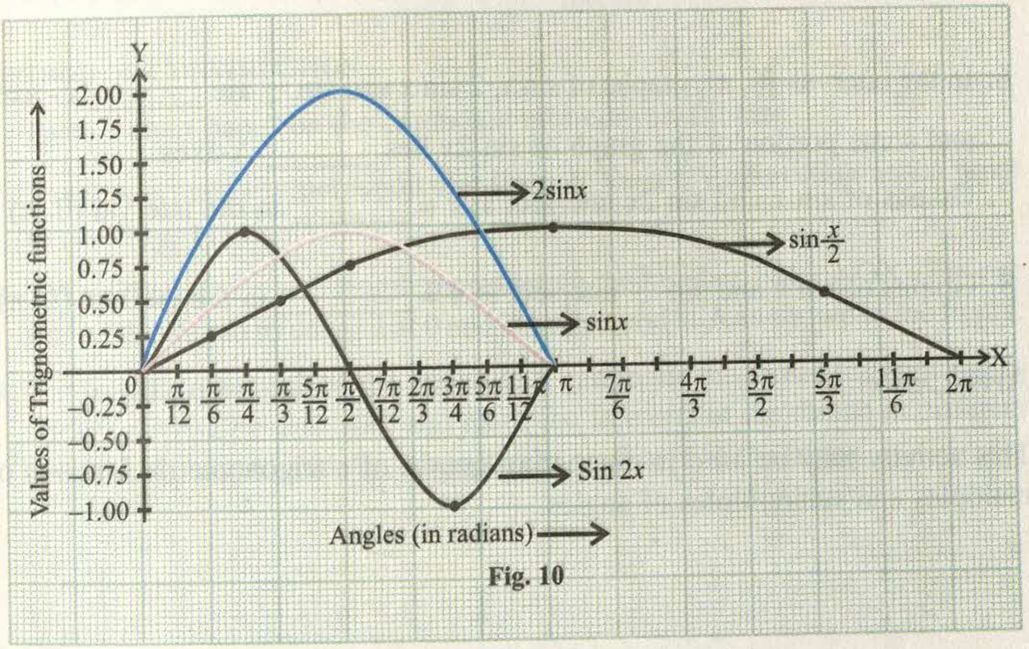


Fig. 10

OBSERVATION

1. Graphs of $\sin x$ and $2 \sin x$ are of same shape but the maximum height of the graph of $\sin x$ is ________ the maximum height of the graph of ________.
2. The maximum height of the graph of $\sin 2x$ is ________. It is at $x =$ ________.
3. The maximum height of the graph of $2 \sin x$ is ________. It is at $x =$ ________.

4. The maximum height of the graph of $\sin \frac{x}{2}$ is __________ . It is at $\frac{x}{2} =$ __________ .

5. At $x =$ ________, $\sin x = 0$, at $x =$ ________, $\sin 2x = 0$ and at $x =$ ________, $\sin\frac{x}{2} = 0$.

6. In the interval $[0, \pi]$, graphs of $\sin x$, $2 \sin x$ and $\sin \frac{x}{2}$ are ________ x - axes and some portion of the graph of $\sin 2x$ lies ________ x-axes.

7. Graphs of $\sin x$ and $\sin 2x$ intersect at $x =$ ______ in the interval $(0, \pi)$

8. Graphs of $\sin x$ and $\sin \frac{x}{2}$ intersect at $x =$ ________ in the interval $(0, \pi)$.

APPLICATION

This activity may be used in comparing graphs of a trigonometric function of multiples and submultiples of angles.

Activity 11

OBJECTIVE

To inerpret geometrically the meaning of $i=\sqrt{-1}$ and its integral powers.

MATERIAL REQUIRED

Cardboard, chart paper, sketch pen, ruler, compasses, adhesive, nails, thread.

METHOD OF CONSTRUCTION

1. Paste a chart paper on the cardboard of a convenient size.
2. Draw two mutually perpendicular lines $X'X$ and $Y'Y$ interesting at the point O (see Fig. 11).
3. Take a thread of a unit length representing the number 1 along OX . Fix one end of the thread to the nail at 0 and the other end at A as shown in the figure.
4. Set free the other end of the thread at A and rotate the thread through angles of 90º, 180º, 270º and 360º and mark the free end of the thread in different cases as A_1, A_2, A_3 and A_4 , respectively, as shown in the figure.

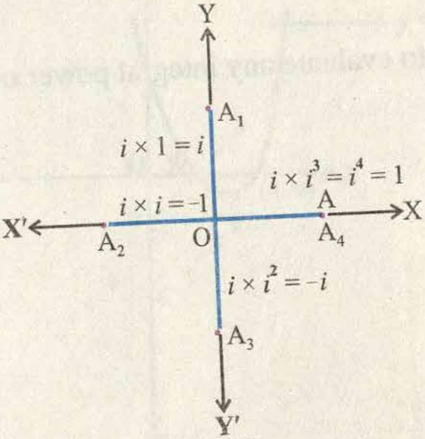


Fig. 11

DEMONSTRATION

1. In the argand plane, OA, OA_1, OA_2, OA_3, OA_4 represent, respectively, 1, i, -1, $-i$, 1.
2. $OA_1 = i = 1 \times i$, $OA_2 = -1 = i \times i = i^2$, $OA_3 = -i = i \times i \times i = i^3$ and so on. Each time, rotation of OA by 90º is equivalent to multiplication by i. Thus, i is referred to as the multiplying factor for a rotation of 90º.

OBSERVATION

1. On rotating OA through 90º, $OA_1 = 1 \times i =$ ________.
2. On rotating OA through an angle of 180°, $OA_2 = 1 \times$ __ $\times$ __ $=$ ______.
3. On rotation of OA through 270º (3 right angles), $OA_3 =$

   $1 \times$ _______ $\times$ ______ $\times$ ______ $=$ ______.
4. On rotating OA through 360º (4 right angles),

   $OA_4 = 1 \times$ _______ $\times$ ______ $\times$ ______ $\times$ ______ $=$ _________.
5. On rotating OA through n-right angles

   $OA_n = 1 \times$ ________ $\times$ _______ $\times$ ______ $\times$ ______ $\times \ldots n$ times $=$ ___________.

APPLICATION

This activity may be used to evaluate any integral power of i.

Activity 12

OBJECTIVE

To obtain a quadratic function with the help of linear functions graphically.

MATERIAL REQUIRED

Plywood sheet, pieces of wires.

METHOD OF CONSTRUCTION

1. Take two wires of equal length.
2. Fix them at O in a plane (on the plywood sheet) at right angle to each other to represent *x*-axis and *y*-axis (see Fig.12)
3. Take a piece of wire and fix it in such a way that it meets the *x*-axis at a distance of *a* units from O in the positive direction and meets *y*-axis at a distance of *a* units below O as shown in the figure. Mark these points as B and A, respectively.

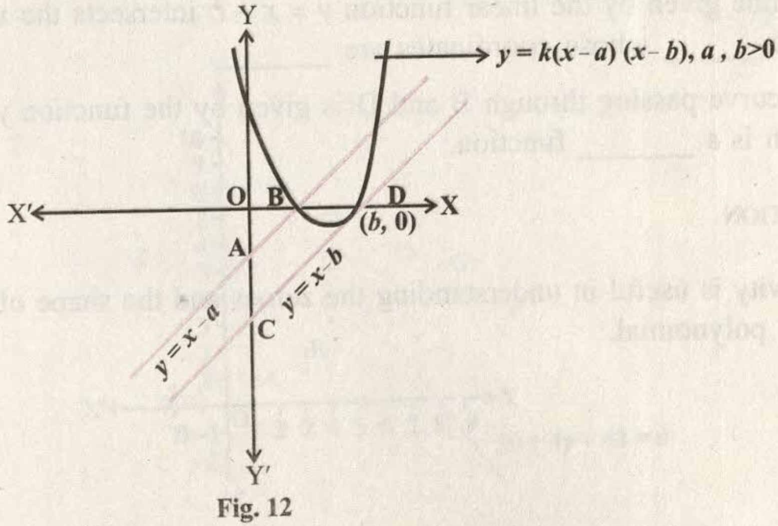


Fig. 12

4. Similarly, take another wire and fix it in such a way that it meets the x-axis at a distance of b units from O in the positive direction and meets y-axis at a distance of b units below O as shown in the Fig.12. Mark these points as D and C, respectively.
5. Take one more wire and fix it in such a way that it passes through the points where straight wires meet the x-axis and the wire takes the shape of a curve (parabola) as shown in the Fig.12.

DEMONSTRATION

1. The wire through the points A and B represents the straight line given by $y = x - a$ intersecting the x and y-axis at $(a, 0)$ and $(0, - a)$, respectively.
2. The wire through the points C and D represents the straight line given by $y = x - b$ intersecting x and y axis at $(b, 0)$ and $(0, - b)$, respectively.
3. The wire through B and D represents a curve given by the function $y = k(x - a)(x - b) = k[x^2 - (a + b)x + ab]$, where k is an arbitrary constant.

OBSERVATION

1. The line given by the linear function $y = x - a$ intersects the x-axis at the point ______ whose coordinates are ________.
2. The line given by the linear function $y = x - b$ intersects the x-axis at the point ______ whose coordinates are _______.
3. The curve passing through B and D is given by the function $y =$ ______, which is a _______ function.

APPLICATION

This activity is useful in understanding the zeroes and the shape of graph of a quadratic polynomial.

Activity 13

OBJECTIVE

To verify that the graph of a given inequality, say $5x + 4y - 40 < 0$, of the form $ax + by + c < 0$, $a, b > 0$, $c < 0$ represents only one of the two half planes.

MATERIAL REQUIRED

Cardboard, thick white paper, sketch pen, ruler, adhesive.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw two perpendicular lines X′OX and Y′OY to represent x-axis and y-axis, respectively.
3. Draw the graph of the linear equation corresponding to the given linear inequality.
4. Mark the two half planes as I and II as shown in the Fig. 13.

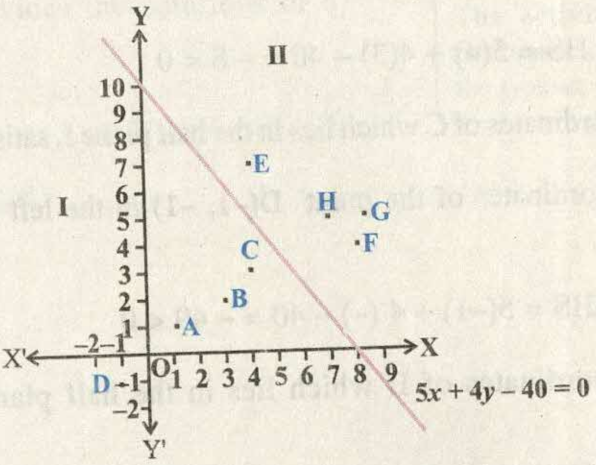


Fig. 13

DEMONSTRATION

1. Mark some points O(0, 0), A(1, 1), B(3, 2), C(4, 3), D(–1, –1) in half plane I and points E(4, 7), F(8, 4), G(9, 5), H(7, 5) in half plane II.

2. (i) Put the coordinates of O (0,0) in the left hand side of the inequality.

   Value of LHS = $5(0) + 4(0) - 40 = -40 < 0$

   So, the coordinates of O which lies in half plane I, satisfy the inequality.

   (ii) Put the coordinates of the point E (4, 7) in the left hand side of the inequality.

   Value of LHS = $5(4) + 4(7) - 40 = 8 \nless 0$ and hence the coordinates of the point E which lie in the half plane II does not satisfy the given inequality.

   (iii) Put the coordinates of the point F(8, 4) in the left hand side of the inequality. Value of LHS = $5(8) + 4(4) - 40 = 16 \nless 0$

   So, the coordinates of the point F which lies in the half plane II do not satisfy the inequality.

   (iv) Put the coordinates of the point C(4, 3) in the left hand side of the inequality.

   Value of LHS = $5(4) + 4(3) - 40 = -8 < 0$

   So, the coordinates of C which lies in the half plane I, satisfy the inequality.

   (v) Put the coordinates of the point D(–1, –1) in the left hand side of the inequality.

   Value of LHS = $5(-1) + 4(-) - 40 = -49 < 0$

So, the coordinates of D which lies in the half plane I, satisfy the inequality.

(iv) Similarly points A (1, 1), lies in a half plane I satisfy the inequality. The points G (9, 5) and H (7, 5) lies in half plane II do not satisfy the inequality.

Thus, all points O, A, B, C, satisfying the linear inequality $5x + 4y - 40 <$ lie only in the half plane I and all the points E, F, G, H which do not satisfy the linear inequality lie in the half plane II.

Thus, the graph of the given inequality represents only one of the two corresponding half planes.

OBSERVATION

Coordinates of the point A _________ the given inequality (satisfy/does not satisfy).

Coordinates of G _________ the given inequality.

Coordinates of H _________ the given inequality.

Coordinates of E are _________ the given inequality.

Coordinates of F _________ the given inequality and is in the half plane___.

The graph of the given inequality is only half plane _______.

APPLICATION

This activity may be used to identify the half plane which provides the solutions of a given inequality.

NOTE

The activity can also be performed for the inequality of the type $ax + by + c > 0$.

Activity 14

OBJECTIVE

To find the number of ways in which three cards can be selected from given five cards.

MATERIAL REQUIRED

Cardboard sheet, white paper sheets, sketch pen, cutter.

METHOD OF CONSTRUCTION

1. Take a cardboard sheet and paste white paper on it.
2. Cut out 5 identical cards of convenient size from the cardboard.
3. Mark these cards as C_1, C_2, C_3, C_4 and C_5.

DEMONSTRATION

1. Select one card from the given five cards.
2. Let the first selected card be C_1. Then other two cards from the remaining four cards can be : C_2C_3, C_2C_4, C_2C_5, C_3C_4, C_3C_5 and C_4C_5.Thus, the possible selections are : $C_1C_2C_3$, $C_1C_2C_4$, $C_1C_2C_5$,$C_1C_3C_4$, $C_1C_3C_5$, $C_1C_4C_5$. Record these on a paper sheet.
3. Let the first selected card be C_2. Then the other two cards from the remaining 4 cards can be : C_1C_3, C_1C_4, C_1C_5, C_3C_4, C_3C_5, C_4C_5.Thus, the possible selections are: $C_2C_1C_3$, $C_2C_1C_4$, $C_2C_1C_5$,$C_2C_3C_4$, $C_2C_3C_5$, $C_2C_4C_5$. Record these on the same paper sheet.
4. Let the first selected card be C_3. Then the other two cards can be : C_1C_2, C_1C_4, C_1C_5, C_2C_4, C_2C_5, C_4C_5.Thus, the possible selections are : $C_3C_1C_2$, $C_3C_1C_4$, $C_3C_1C_5$,$C_3C_2C_4$, $C_3C_2C_5$, $C_3C_4C_5$. Record them on the same paper sheet.
5. Let the first selected card be C_4. Then the other two cards can be : C_1C_2, C_1C_3, C_2C_3, C_1C_5, C_2C_5, C_3C_5 Thus, the possible selections are: $C_4C_1C_2$, $C_4C_1C_3$,$C_4C_2C_3$, $C_4C_1C_5$, $C_4C_2C_5$, $C_4C_3C_5$. Record these on the same paper sheet.

6. Let the first selected card be C_5. Then the other two cards can be: C_1C_2, C_1C_3, C_1C_4, C_2C_3, C_2C_4, C_3C_4 Thus, the possible selections are: $C_5C_1C_2$,$C_5C_1C_3$, $C_5C_1C_4$, $C_5C_2C_3$, $C_5C_2C_4$, $C_5C_3C_4$. Record these on the same paper sheet.

7. Now look at the paper sheet on which the possible selectios are listed. Here, there are in all 30 possible selections and each of the selection is repeated thrice. Therefore, the number of distinct selection $= 30 \div 3 = 10$ which is same as $5C_3$.

OBSERVATION

1. $C_1C_2C_3$, $C_2C_1C_3$ and $C_3C_1C_2$ represent the _______ selection.
2. $C_1C_2C_4$, _____________, ________ represent the same selection.
3. Among $C_2C_1C_5$, $C_1C_2C_5$, $C_1C_2C_3$, ________ and ________ represent the same selection.
4. $C_2C_1C_5$, $C_1C_2C_3$, represent _______ selections.
5. Among $C_3C_1C_5$, $C_1C_4C_3$, $C_5C_3C_4$, $C_4C_2C_5$, $C_2C_4C_3$, $C_1C_3C_5$

   $C_3C_1C_5$, _______ represent the same selections.

   $C_3C_1C_5$, $C_1C_4C_5$, ______, _____, represent different selections.

APPLICATION

Activities of this type can be used in understanding the general formula for finding the number of possible selections when r objects are selected from given n distinct objects, i.e., ${}^{n}C_r = \dfrac{n!}{r!(n-r)!}$.

Activity 15

OBJECTIVE

To construct a Pascal's Triangle and to write binomial expansion for a given positive integral exponent.

MATERIAL REQUIRED

Drawing board, white paper, matchsticks, adhesive.

METHOD OF CONSTRUCTION

1. Take a drawing board and paste a white paper on it.
2. Take some matchsticks and arrange them as shown in Fig.15.

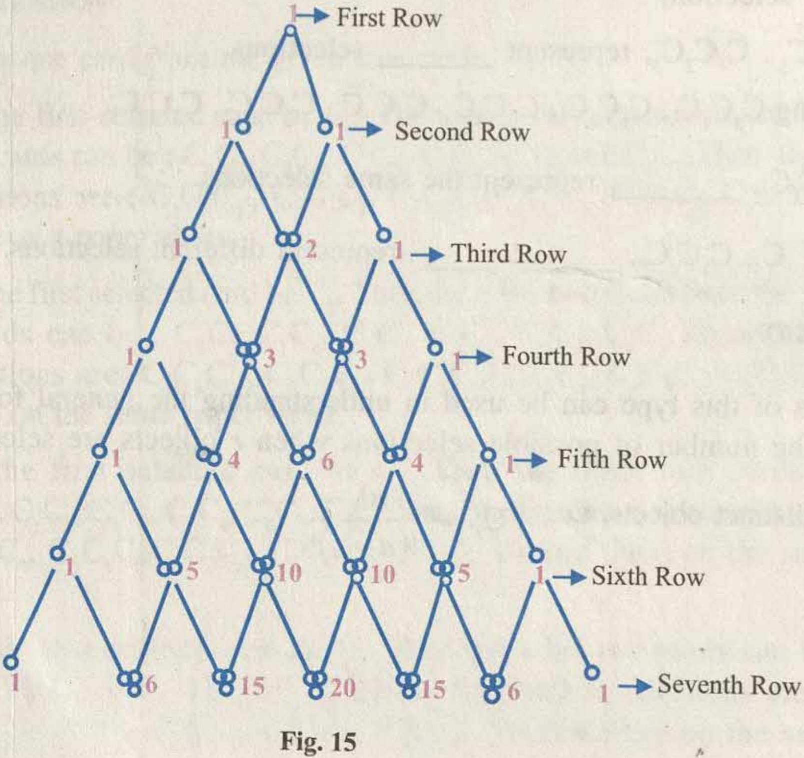


Fig. 15

3. Write the numbers as follows:

    1 (first row)
    1 1 (second row)
    1 2 1 (third row)
    1 3 3 1 (fourth row), 1 4 6 4 1 (fifth row) and so on (see Fig. 15).

4. To write binomial expansion of $(a + b)^n$, use the numbers given in the $(n + 1)^{\text{th}}$ row.

DEMONSTRATION

1. The above figure looks like a triangle and is referred to as Pascal's Triangle.
2. Numbers in the second row give the coefficients of the terms of the binomial expansion of $(a + b)^1$. Numbers in the third row give the coefficients of the terms of the binomial expansion of $(a + b)^2$, numbers in the fourth row give coefficients of the terms of binomial expansion of $(a + b)^3$. Numbers in the fifth row give coefficients of the terms of binomial expansion of $(a + b)^4$ and so on.

OBSERVATION

1. Numbers in the fifth row are __________, which are coefficients of the binomial expansion of __________.
2. Numbers in the seventh row are ____________, which are coefficients of the binomial expansion of _______.
3. $(a + b)^3 = \underline{\quad}\, a^3 + \underline{\quad}\, a^2b + \underline{\quad}\, ab^2 + \underline{\quad}\, b^3$
4. $(a + b)^5 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}.$
5. $(a + b)^6 = \underline{\quad}\, a^6 + \underline{\quad}\, a^5b + \underline{\quad}\, a^4b^2 + \underline{\quad}\, a^3b^3 + \underline{\quad}\, a^2b^4 + \underline{\quad}\, ab^5 + \underline{\quad}\, b^6.$
6. $(a + b)^8 = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}.$
7. $(a + b)^{10} = \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad} + \underline{\quad}.$

APPLICATION

The activity can be used to write binomial expansion for $(a + b)^n$, where n is a positive integer.

Activity 16

OBJECTIVE

To obtain formula for the sum of squares of first *n*-natural numbers.

MATERIAL REQUIRED

Wooden/plastic unit cubes, coloured papers, adhesive and nails.

METHOD OF CONSTRUCTION

1. Take 1 (= 1^2) wooden/plastic unit cube Fig.16.1.
2. Take 4 (= 2^2) wooden/plastic unit cubes and form a cuboid as shown in Fig.16.2.
3. Take 9 (= 3^2) wooden/plastic unit cubes and form a cuboid as shown in Fig.16.3.
4. Take 16 (= 4^2) wooden/plastic unit cubes and form a cuboid as shown in Fig. 16.4 and so on.
5. Arrange all the cube and cuboids of Fig. 16.1 to 16.4 above so as to form an echelon type structure as shown in Fig.16.5.
6. Make six such echelon type structures, one is already shown in Fig. 16.5.
7. Arrange these five structures to form a bigger cuboidal block as shown in Fig. 16.6.

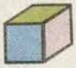


Fig 16.1

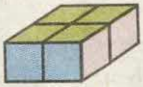


Fig 16.2

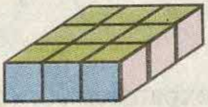


Fig 16.3

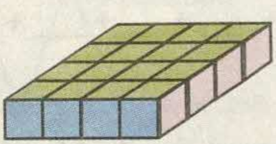


Fig 16.4

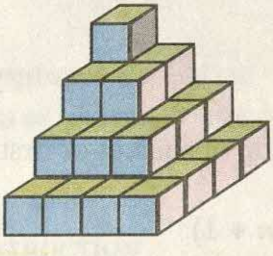


Fig. 16.5

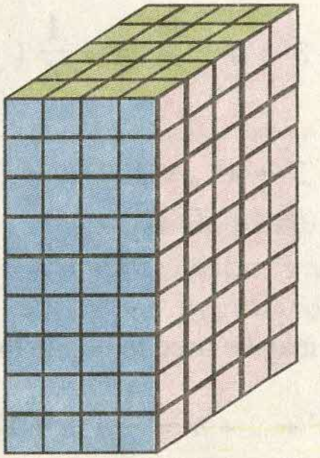


Fig. 16.6

DEMONSTRATION

1. Volume of the structure as given in Fig. 16.5

   $= (1 + 4 + 9 + 16)$ cubic units $= (1^2 + 2^2 + 3^2 + 4^2)$ cubic units.

2. Volume of 6 such structures $= 6\,(1^2 + 2^2 + 3^2 + 4^2)$ cubic units.
3. Volume of the cuboidal block formed in Fig. 16.6 (which is cuboid of dimensions $= 4 \times 5 \times 9) = 4 \times (4 + 1) \times (2 \times 4 + 1)$.
4. Thus, $6\,(1^2 + 2^2 + 3^2 + 4^2) = 4 \times (4 + 1) \times (2 \times 4 + 1)$

   i.e., $$1^2+2^2+3^2+4^2=\frac{1}{6}[4\times(4+1)\times(2\times4+1)]$$

OBSERVATION

1. $1^2 + 2^2 + 3^2 + 4^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

2. $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

3. $1^2 + 2^2 + 3^2 + 4^2 + \ldots + 10^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

4. $1^2 + 2^2 + 3^2 + 4^2 \ldots + 25^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

5. $1^2 + 2^2 + 3^2 + 4^2 \ldots + 100^2 = \frac{1}{6}$ (_____) × (_____) × (_____).

APPLICATION

This activity may be used to obtain the sum of squares of first n natural numbers

as $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{1}{6} n\,(n + 1)\,(2n + 1)$.

Activity 17

OBJECTIVE

An alternative approach to obtain formula for the sum of squares of first n natural numbers.

MATERIAL REQUIRED

Wooden/plastic unit squares, coloured pencils/sketch pens, scale.

METHOD OF CONSTRUCTION

1. Take unit squares, 1, 4, 9, 16, 25 ... as shown in Fig. 17.1 and colour all of them with (say) Black colour.

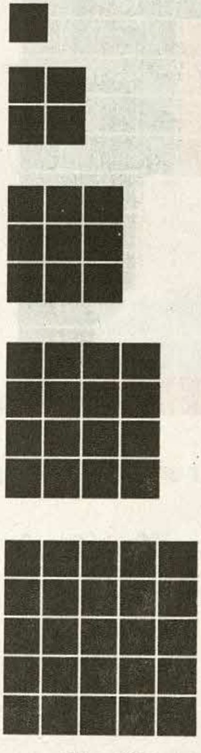


Fig. 17.1

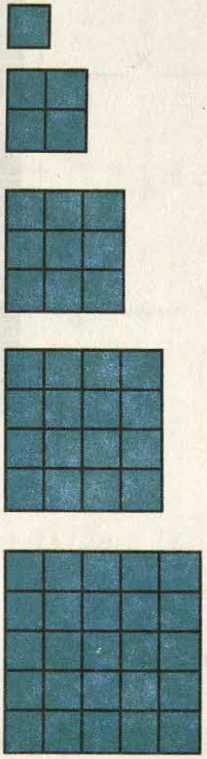


Fig. 17.2

2. Take another set of unit squares 1, 4, 9, 16, 25 ... as shown in Fig. 17.2 and colour all of them with (say) green colour.
3. Take a third set of unit squares 1, 4, 9, 16, 25 ... as shown in Fig. 17.3 and colour unit squares with different colours.
4. Arrange these three set of unit squares as a rectangle as shown in Fig. 17.4.

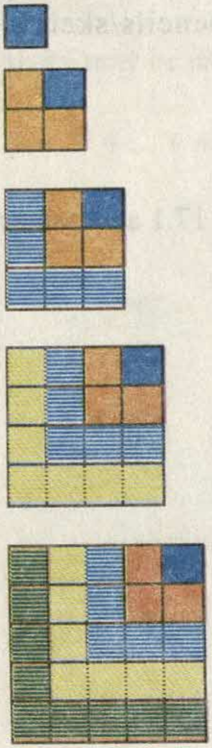


Fig. 17.3

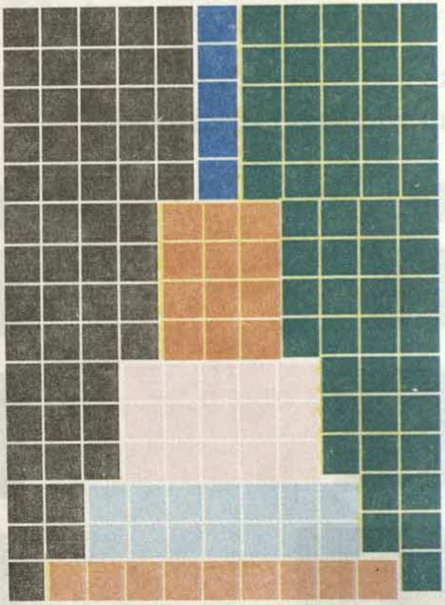


Fig. 17.4

DEMONSTRATION

1. Area of one set as given in Fig. 17.1

$$= (1 + 4 + 9 + 16 + 25) \text{ sq. units}$$

$$= (1^2 + 2^2 + 3^2 + 4^2 + 5^2) \text{ sq. units.}$$

2. Area of three such sets = $3\,(1^2 + 2^2 + 3^2 + 4^2 + 5^2)$

3. Area of rectangle $= 11 \times 15 = [2\,(5) + 1]\left[\dfrac{5\times 6}{2}\right]$

$$\therefore \quad 3\,(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = \frac{1}{2}\,[5 \times 6]\,[2\,(5) + 1]$$

$$\text{or} \quad 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6}\,[5 \times (5 + 1)]\,[2\,(5) + 1].$$

OBSERVATION

$$3\,(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = \frac{1}{2}\,(\_\_\_\_ \times \_\_\_\_)\,(\_\_\_ + 1)$$

$$\Rightarrow \quad 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{1}{6}\,(\_\_\_\_ \times \_\_\_\_)\,(\_\_\_ + 1)$$

$$\therefore \quad 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 = \frac{1}{6}\,(\_\_\_\_ \times \_\_\_\_)\,(\_\_\_ + 1)$$

$$1^2 + 2^2 + 3^2 + 4^2 + ... + 10^2 = \frac{1}{6}\,(\_\_\_\_ \times \_\_\_\_)\,(\_\_\_ + 1).$$

APPLICATION

This activity may be used to establish

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{1}{6}\,n\,(n + 1)\,(2n + 1).$$

Activity 18

OBJECTIVE

To demonstrate that the Arithmetic mean of two different positive numbers is always greater than the Geometric mean.

MATERIAL REQUIRED

Coloured chart paper, ruler, scale, sketch pens, cutter.

METHOD OF CONSTRUCTION

1. From chart paper, cut off four rectangular pieces of dimension $a \times b$ $(a > b)$.
2. Arrange the four rectangular pieces as shown in figure. 18.

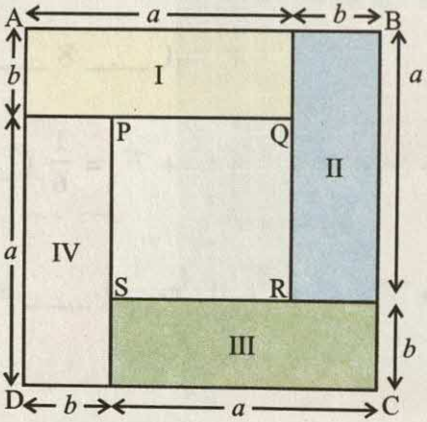


Fig. 18

DEMONSTRATION

1. ABCD is a square of side $(a + b)$ units.
2. Area ABCD = $(a + b)^2$ sq. units.
3. Area of four rectangular pieces = $4\,(ab)$ = $4ab$ sq. units.

4. PQRS is a square of side $(a - b)$ units.
5. Area ABCD = Sum of the areas of four rectangular pieces + area of square PQRS.

$\therefore$ Area ABCD > sum of the areas of four rectangular pieces

i.e., $(a + b)^2 > 4ab$

or $\left(\frac{a+b}{2}\right)^2 > ab$

$\therefore$ $\frac{a+b}{2} > \sqrt{ab}$, i.e., A.M. > G.M.

OBSERVATION

Take $a = 5\text{cm}$, $b = 3\text{cm}$

$\therefore$ AB = $a + b$ = ________ units.

Area of ABCD = $(a + b)^2$ = ________ sq. units.

Area of each rectangle = ab = ________ sq. units.

Area of square PQRS = $(a - b)^2$ = ________ sq. units.

Area ABCD = 4 (area of rectangular piece) + Area of square PQRS

________ = 4 (________) + (________)

$\therefore$ ________ > 4 (________)

i.e. $(a + b)^2 > 4ab$ or $\left(\frac{a+b}{2}\right)^2 > ab$

or $\frac{a+b}{2} > \sqrt{ab}$ $\therefore$ AM > GM

Activity 19

OBJECTIVE

To establish the formula for the sum of the cubes of the first n natural numbers.

MATERIAL REQUIRED

Thermocol sheet, thermocol balls, pins, pencil, ruler, adhesive, chart paper, cutter.

METHOD OF CONSTRUCTION

1. Take (or cut) a square sheet of thermocol of a convenient size and paste a chart paper on it.
2. Draw horizontal and vertical lines on the pasted chart paper to form 225 small squares as shown in Fig. 19.

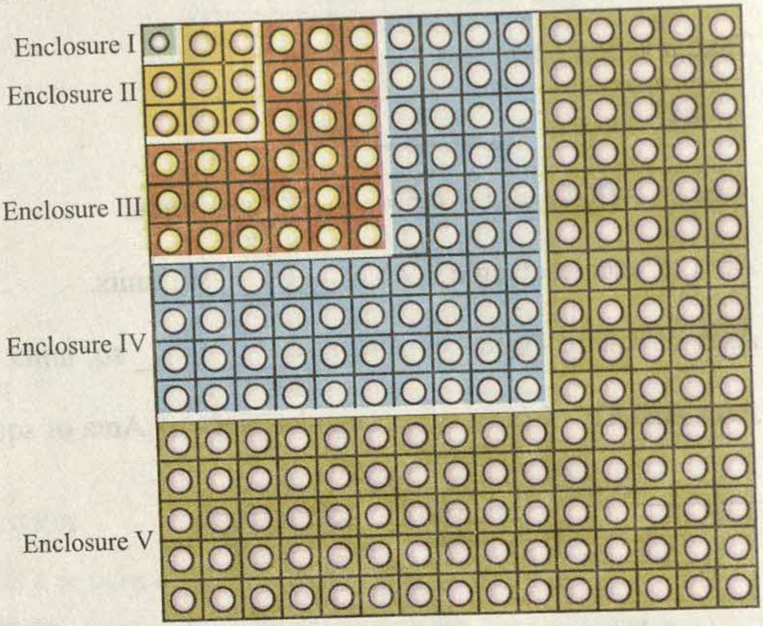


Fig. 19

3. Fix a thermocol ball with the help of a pin at the square on the upper left most corner.

4. Fix 2^3, i.e., 8, thermocol balls with the help of 8 pins on the same square sheet in 8 squares adjacent to the previous square as shown in the figure.
5. Fix 3^3, i.e., 27 thermocol balls with the help of 27 pins on the same square sheet in 27 squares adjacent to the previous 8 squares.
6. Continue fixing the thermocol balls in this way till all the squares are filled (see. Fig. 19).

DEMONSTRATION

1. Number of balls in Enclosure I $=1^3=1=\left(\frac{1\times 2}{2}\right)^2$.
2. Number of balls in Enclosure II $=1^3+2^3=9=\left(\frac{2\times 3}{2}\right)^2$.
3. Number of balls in Enclosure III $= 1^3 + 2^3 + 3^3 = 36 = \left(\frac{3\times 4}{2}\right)^2$.
4. Number of balls in Enclosure IV $= 1^3 + 2^3 + 3^3 + 4^3 = 100 = \left(\frac{4\times 5}{2}\right)^2$.
5. Total number of balls in Enclosure V $= 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

   $= 225 = \left(\frac{5\times 6}{2}\right)^2$.

OBSERVATION

By actual counting of balls

1. Number of balls in Enclosure I $=1^3=$ _______ $=\left(\frac{1\times 2}{2}\right)^2$.

2. Number of balls in Enclosure II = $=13^3+2^3=\underline{\hspace{2cm}}=\left(\frac{\underline{\hspace{1cm}}\times\underline{\hspace{1cm}}}{\underline{\hspace{1cm}}}\right)^2.$

3. Number of balls in Enclosure III

$$=1^3+2^3+\underline{\hspace{2cm}}=\underline{\hspace{2cm}}=\left(\frac{\underline{\hspace{1cm}}\times\underline{\hspace{1cm}}}{2}\right)^2.$$

4. Number of balls in Enclosure IV

$$=1^3+2^3+(\underline{\hspace{0.5cm}})^3+(\underline{\hspace{0.5cm}})^3=\underline{\hspace{1cm}}=\left(\frac{\underline{\hspace{1cm}}\times\underline{\hspace{1cm}}}{2}\right)^2.$$

5. Number of balls in Enclosure V

$$=(\underline{\hspace{0.5cm}})^3+(\underline{\hspace{0.5cm}})^3+(\underline{\hspace{0.5cm}})^3+(\underline{\hspace{0.5cm}})^3+(\underline{\hspace{0.5cm}})^3=\underline{\hspace{1cm}}=\left(\frac{\underline{\hspace{1cm}}\times\underline{\hspace{1cm}}}{2}\right)^2.$$

APPLICATION

This result can be used in finding the sum of cubes of first *n* natural numbers, i.e.,

$$1^3 + 2^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Activity 20

OBJECTIVE

To verify that the equation of a line passing through the point of intersection of two lines $a_1x + b_1y + c_1=0$ and $a_2x + b_2y + c_2 = 0$ is of the form $(a_1x + b_1y + c_1) + \lambda \; (a_2x + b_2y + c_2) = 0$.

MATERIAL REQUIRED

Cardboard, sketch pen, white paper, adhesive, pencil, ruler.

METHOD OF CONSTRUCTION

1. Take a cardboard of convenient size and paste a white paper on it.
2. Draw two perpendicular lines X′OX and Y′OY on the graph paper. Take same scale for marking points on *x* and *y*-axes.
3. Draw the graph of the given two intersecting lines and note down the point of intersection, say (*h*, *k*) (see Fig. 20.1)

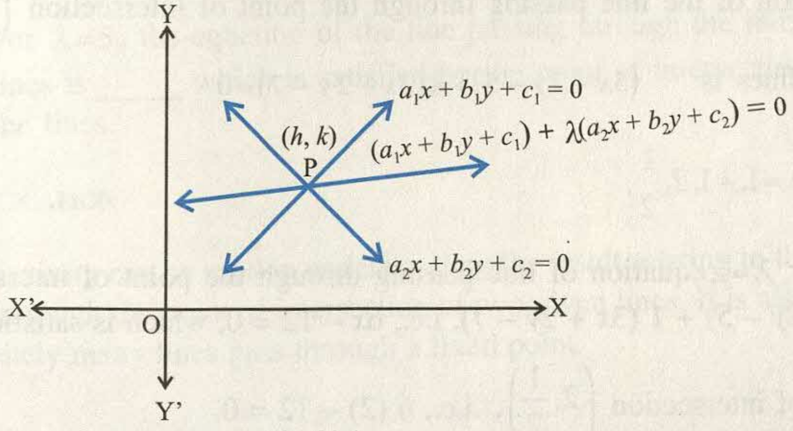


Fig. 20.1

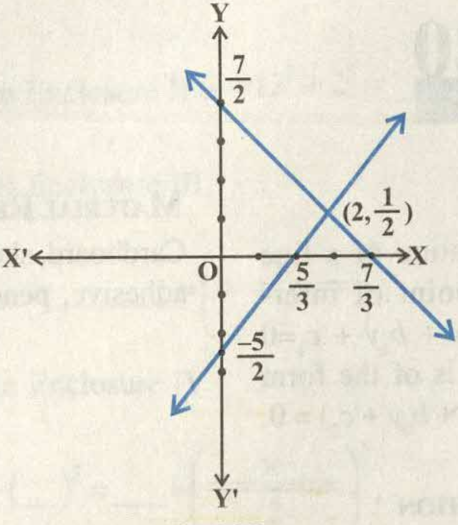


Fig. 20.2

DEMONSTRATION

1. Let the equations of the lines be $3x - 2y = 5$ and $3x + 2y = 7$.

2. The point of intersection of these lines is $\left(2, \frac{1}{2}\right)$ (See Fig. 20.2).

3. Equation of the line passing through the point of intersection $\left(2, \frac{1}{2}\right)$ of these lines is $(3x - 2y - 5) + \lambda(3x + 2y - 7) = 0$ (1)

4. Take $\lambda = 1, -1, 2, \frac{1}{2}$.

5. (i) For $\lambda = 1$, equation of line passing through the point of intersection is $(3x - 2y - 5) + 1\,(3x + 2y - 7)$, i.e., $6x - 12 = 0$, which is satisfied by the point of intersection $\left(2, \frac{1}{2}\right)$, i.e., $6\,(2) - 12 = 0$

(ii) For $\lambda = -1$, the equation of line passing through the point of intersection is

$(3x - 2y - 5) - 1\ (3x + 2y - 7) = 0$ is $-4y + 2 = 0$, which is also satisfied by the point of intersection $\left(2, \frac{1}{2}\right)$.

(iii) For $\lambda = 2$, the equation is $(3x - 2y - 5) + 2\ (3x + 2y - 7) = 0$, i.e., $9x + 2y - 19 = 0$, which is again satisfied by the point $\left(2, \frac{1}{2}\right)$.

OBSERVATION

1. For $\lambda = 3$, the equation of the line passing through intersection of the lines is _________ which is satisfied by the point $\left(2, \frac{1}{2}\right)$.
2. For $\lambda = 4$, the equation of the line passing through point of the intersection of the lines is ________ which is satisfied by the point of intersection __________ of the lines.
3. For $\lambda = 5$, the equation of the line passing through the intersection of the lines is _____ which is satisfied by the point of intersection ________ of the lines.

APPLICATION

The activity can be used in understanding the result relating to the equation of a line through the point of intersection of two given lines. It is also observed that infinitely many lines pass through a fixed point.

Activity 21

OBJECTIVE

To construct different types of conic sections.

MATERIAL REQUIRED

Transparent sheet, scissors, hardboard, adhesive, white paper.

METHOD OF CONSTRUCTION

1. Take a hardboard of convenient size and paste a white paper on it.
2. Cut a transparent sheet in the shape of sector of a circle and fold it to obtain a right circular cone as shown in Fig.21.1.
3. Form 4 more such cones of the same size using transparent sheet. Put these cones on a hardboard.
4. Cut these cones with a transparent plane sheet in different positions as shown in Fig. 21.2 to Fig. 21.5.

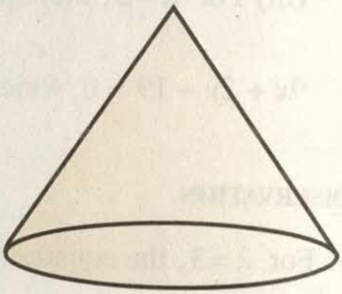


Fig 21.1

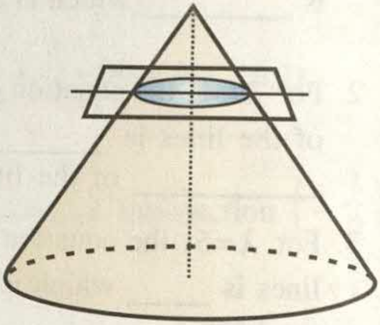


Fig 21.2

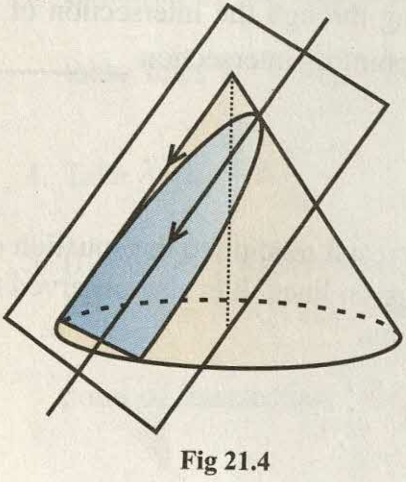


Fig 21.4

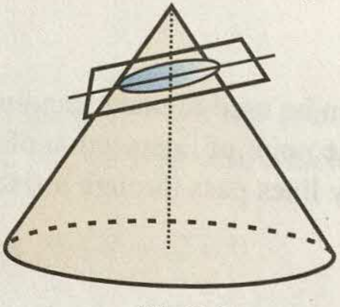


Fig 21.3

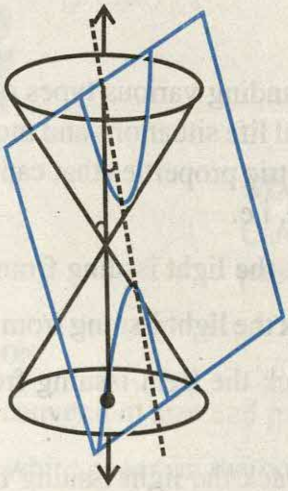


Fig 21.5

DEMONSTRATION

1. In Fig. 21.2, the transparent plane sheet cuts the cone in such a way that the sheet is parallel to the base of the cone. The section so obtained is a circle.
2. In Fig. 21.3, the plane sheet is inclined slightly to the axes of the cone. The section so obtained is an ellipse.
3. In Fig. 21.4, the plane sheet is parallel to a generator (slant height) of the cone. The section so obtained is a parabola.
4. In Fig. 21.5 the plane is parallel to the axis of the cone. The sections so obtained is a part of a hyperbola.

OBSERVATION

1. In Fig. 21.2, the transparent plane sheet is ______ to the base of the cone. The section obtained is _____.
2. In Fig. 21.3, the plane sheet is inclined to ______. The conic section obtained is _________.
3. In Fig. 21.4, the plane sheet is parallel to the _________. The conic section so obtained is ______.
4. In Fig. 21.5, the plane sheet is _______ to the axis. The conic section so obtained is a part of ______.

APPLICATION

This activity helps in understanding various types of conic sections which have wide spread applications in real life situations and modern sciences. For example, conics have interesting geometric properties that can be used for the reflection of light rays and beams of sound, i.e.

1. Circular disc reflects back the light issuing from centre to the centre again.
2. Elliptical disc reflects back the light issuing from one focus to the other focus.
3. Parabolic disc reflects back the light issuing from one focus parallel to its axis.
4. Hyperbolic disc reflects back the light issuing from one focus as if coming from other focus.

Activity 22

OBJECTIVE

To construct a parabola.

MATERIAL REQUIRED

Cardboard, white paper, sketch pen, pencil, compass, ruler etc.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Mark a point S on the white paper on the board (see Fig. 22).
3. Through S draw a line. Draw another line *l* perpendicular to the line through S at some distance *k* units to the left of S.

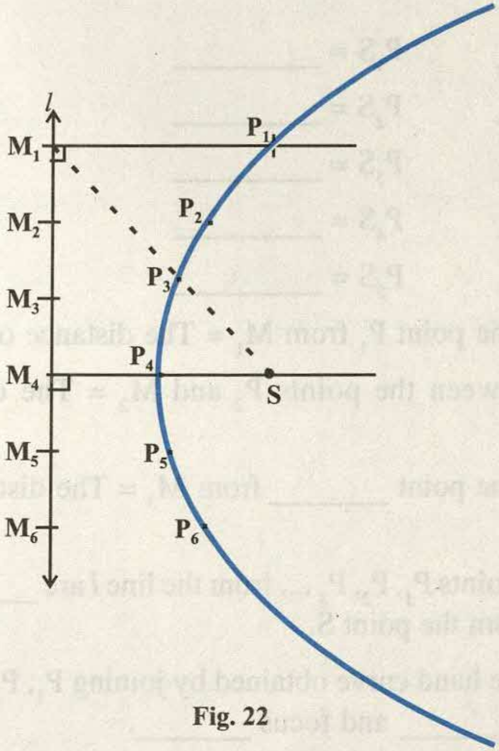


Fig. 22

4. Take any point M_1 on the line l. Draw the perpendicular to l at this point.
5. Join M_1S and draw perpendicular bisector of M_1S meeting the perpendicular through M_1 at the point P_1.
6. Take another point M_2 on the line l and repeat the process as explained in (5) above to obtain the point P_2.
7. Take some more points M_3, M_4, M_5, ... on the line l and repeat the above process to obtain points P_3, P_4, P_5, ..., respectively.
8. Draw a free hand curve through the points P_1, P_2, P_3, P_4, ... (see Fig. 22)

DEMONSTRATION

The points P_1, P_2, P_3, ... are such that the distance of each point from the fixed point S is same as the distance of the point from the line l. So, the free hand curve drawn through these points is a parabola with focus S and directrix l.

OBSERVATION

1. P_1M_1 = ________ P_1S = ________
2. P_2M_2 = ________ P_2S = ________
3. P_3M_3 = ________ P_3S = ________
4. P_4M_4 = ________ P_4S = ________
5. P_5M_5 = ________ P_5S = ________
6. The distance of the point P_1 from M_1 = The distance of P_1 from _______.
7. The distance between the points P_2 and M_2 = The distance of P_2 from _______.

   The distance of the point _______ from M_3 = The distance of the point P_3 from ______.
8. Distances of the points P_1, P_2, P_3 .... from the line l are ______ to the distances of these points from the point S.
9. Therefore, the free hand curve obtained by joining P_1, P_2, P_3, ... is a _______ with directrix _________ and focus _______.

10. Distance of the vertex P_4 and S = ______.

11. Distance of the vertex of parabola from the directrix = ______.

APPLICATION

1. This activity is useful in understanding the terms related to parabola, like directrix, focus, property of the point on the parabola.
2. Parabolas have applications in Science and Engineering.

Activity 23

OBJECTIVE

An alternative method of constructing a parabola.

MATERIAL REQUIRED

Cardboard, white paper, sketch pen, pencil, compasses, ruler, nails, thread.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Take any point S on the white paper fixed on the cardboard.
3. Draw a line through S.
4. Draw another line l perpendicular to the line through S at a distance of k units to the left of S. Let the two lines meet at the point C.

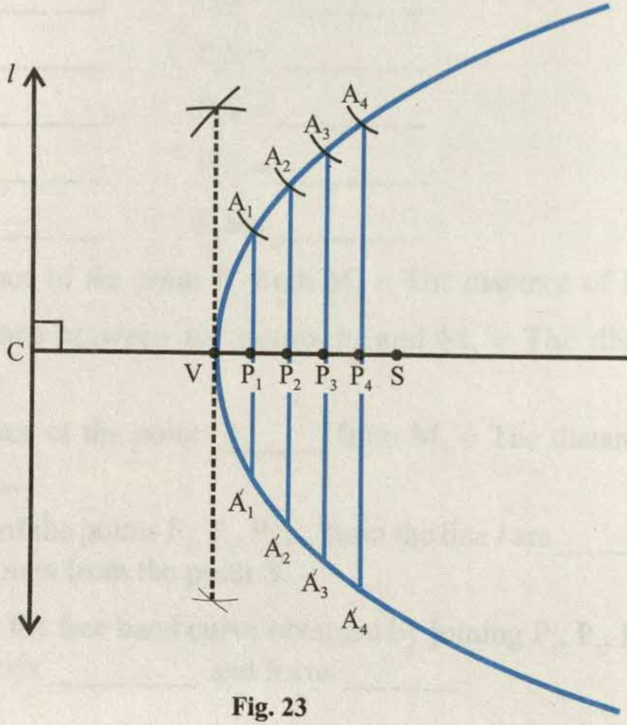


Fig. 23

5. Bisect CS at the point V.
6. Mark the points P_1, P_2, P_3, ... on VS and draw perpendiculars through these points as shown in the Fig. 23.
7. Take S as centre and radius equal to CP_1, draw an arc cutting the perpendicular through P_1 at the point A_1 and A_1'. Similarly with S as the centre and CP_2 as radius, obtain points A_2 and A_2'. Repeat this process for some more points P_3, P_4, ... and obtain points A_3 and A_3', A_4 and A_4';...
8. Fix nails at these points, i.e., $A_1, A_2, \ldots A_1', A_2', \ldots$ and join the foot of the nails by a thread to get a curve as shown in the figure.

DEMONSTRATION

Distance of the point A_1 from $l = CP_1 = SA_1$
Similarly, distance of the point A_2 from $l = CP_2 = SA_2$
distance of the point A_3 from $l = CP_3 = SA_3$ and so on.

Also distance of the point A_1' from $l = CP_1 = SA_1'$

distance of the point A_2' from $l = CP_2 = SA_2'$ and so on.

Thus, every point on the curve is equidistant from the line l and the point S. So, the curve is a parabola, with focus S and directrix l.

OBSERVATION

By actual measurement

1. Distance of A_1 from l = _________, A_1S = _________.
2. Distance of A_2 from l = _________, A_2S = _________.
3. Distance of A_3 from l = _________, A_3S = _________.
4. Distance of A_4 from l = _________, A_4S = _________.
5. Distance of A_1' from l = _________, $A_1'S$ = _________.

6. Distance of A_2' from l = __________, $A_2'S$ = __________.

7. Distance of A_3' from l = __________, $A_3'S$ = __________.

8. Distance of A_4' from l = __________, $A_4'S$ = __________.

9. Distance of any point on the curve from l = Distance of the point from ________.

10. So, the curve is _______ with directrix ________ and focus _________.

APPLICATION

1. This activity is useful in understanding the terms related to a parabola, such as directrix and focus of the parabola.

Activity 24

OBJECTIVE

To construct an ellipse using a rectangle.

MATERIAL REQUIRED

A hardboard, white paper, coloured paper, nails, nylon wire/thread, ruler, adhesive.

METHOD OF CONSTRUCTION

1. Take a rectangular hardboard of a convenient size and paste a white paper on it.
2. Cut a rectangle MNBL of suitable dimensions from a coloured paper and paste it on the hardboard.
3. Divide this rectangle into four congruent rectangles as shown in the Fig.24.

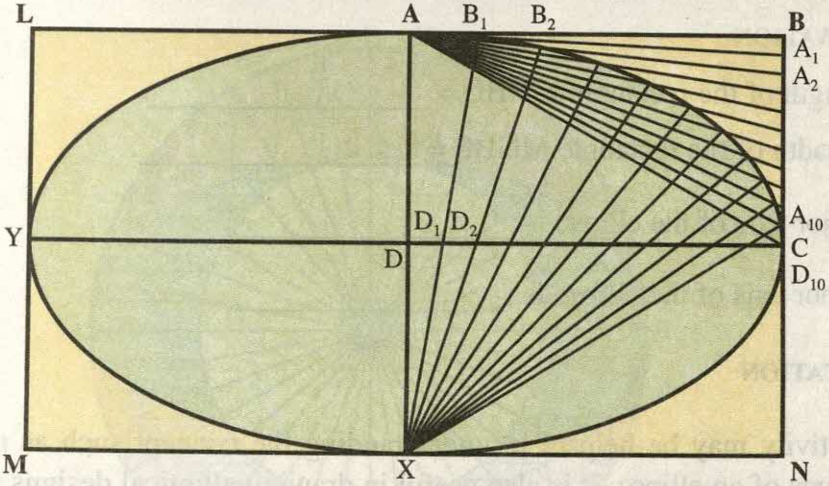


Fig. 24

4. Divide each of the sides BC and DC of the rectangle ADCB, into some equal parts, (say, 11)
5. Mark the point of subdivisions of BC as A_1, A_2, ... and that of DC as D_1, D_2, (See Fig. 24)

6. Join the point A to points, A_1, A_2, ... and draw the lines joining the point X to D_1, D_2, ... (See Fig. 24)
7. Mark the point of intersection of AA_1 and XD_1 as B_1, AA_2 and XD_2 as B_2 and so on.
8. Fix nails at the points B_1, B_2, ..., B_{10}.
9. Join the feet of nails with a nylon wire/thread, as shown in the figure.
10. Repeat the same activity for remaining three congruent rectangles and obtain a curve as shown in Fig. 24.

DEMONSTRATION

The curve obtained looks like an ellipse. The major axis of this ellipse is the length of the rectangle MNBL and the minor axis of the ellipse is the breadth of the rectangle.

OBSERVATION

1. Length of the rectangle MNBL = _______________.
2. Breadth of the rectangle MNBL = _______________.
3. Major axis of the ellipse is _______________.
4. Minor axis of the ellipse is _______________.

APPLICATION

This activity may be helpful in understanding the concept such as major and minor axis of an ellipse. It is also useful in drawing elliptical designs such as in swimming pools, tables, etc.

Activity 25

OBJECTIVE

To construct an ellipse with given major and minor axes.

MATERIAL REQUIRED

A hardboard, white paper, nylon wire/thread, adhesive, chart paper.

METHOD OF CONSTRUCTION

1. Take a rectangular sheet of a hardboard of a convenient size and paste a white paper on it.
2. Mark a point O on it and draw two concentric circles with centre O and radii as given semi-major and semi-minor axis of the ellipse. Mark one of the diameter of bigger circle as AOB and call it a horizontal line (see Fig. 25)

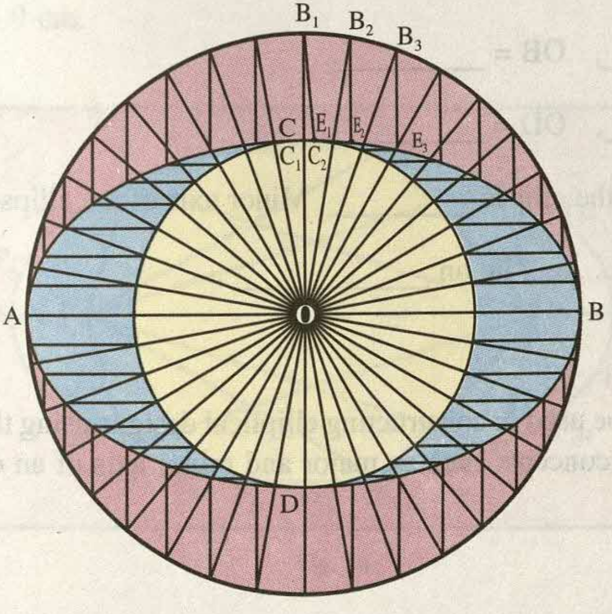


Fig. 25

3. Draw radii of the circles in such a way that the angle between two consecutive radii is the same, say 10º.

4. Take any radius OB_1 of the bigger circle cutting the smaller circle at C_1. Draw a horizontal line through C_1 and draw a perpendicular (vertical line) from B_1 to this horizontal line and obtain point E_1 (see Fig. 25).
5. Repeat this process for all the radii OB_2, OB_3, and so on of the bigger circle and obtain the points E_2, E_3, ... and so on.
6. Fix the nails at the points E_1, E_2, E_3, ... and join the feet of the nails by a nylon wire/thread and obtain a curve (see Fig. 25).

DEMONSTRATION

1. The curve obtained looks like an ellipse.
2. Major axis of the ellipse is AOB and the minor axis of the ellipse is COD, where COD is the diameter of the smaller circle perpendicular to diameter AOB.

OBSERVATION

1. OA = _______, OB = _________.

2. OC = _______, OD = _________.

3. Major axis of the ellipse _________, Minor axis of the ellipse = ________.

4. Points E_1, E_2, ... lie on ___________ .

APPLICATION

This activity may be used in constructing elliptical designs using thread work and also in explaining concepts such as major and minor axis of an ellipse.

Activity 26

OBJECTIVE

To construct an ellipse when two fixed points are given.

MATERIAL REQUIRED

Rectangular cardboard, coloured chart paper, nails, strings, pen, pencil.

METHOD OF CONSTRUCTION

1. Take a rectangular cardboard and paste a chart paper on it.
2. Draw a horizontal line on the chart paper and mark two fixed points F_1 and F_2 on it such that the distance between them is (say) 6 cm. Fix two nails at the points F_1 and F_2.
3. Take a string whose length is more than the distance between the two fixed points (say) 9 cm.

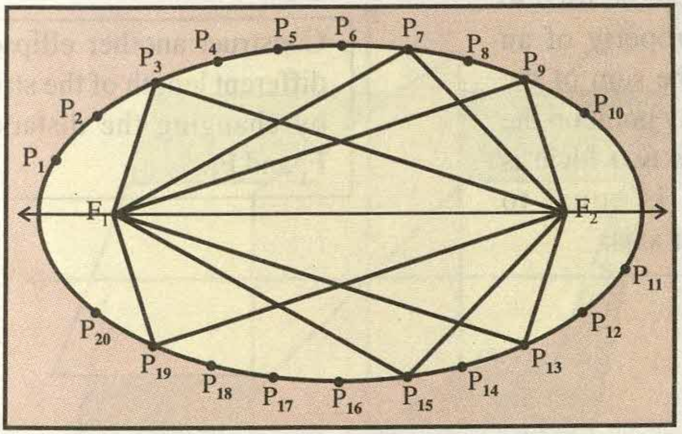


Fig. 26

DEMONSTRATION

1. Fix the two ends of the string at the two nails at F_1 and F_2.

2. With a pencil, stretch the string in the loop without slack and mark at least 10 points P_1, P_2, P_3, ..., etc., on both sides of the line segment joining F_1 and F_2.
3. Join all the points P_i, $i = 1, 2, \ldots 20$ to form an ellipse.

OBSERVATION

1. $P_1F_1 + P_1F_2 =$ ____ .
2. $P_2F_1 + P_2F_2 =$ ____ .
3. $P_3F_1 + P_3F_2 =$ ___, $P_4F_1 + P_4F_2 =$ ___, $P_6F_1 + P_6F_2 =$ ___, $P_9F_1 + P_9F_2 =$ ___.
4. $P_3F_1 + P_3F_2 =$ ____ $+ P_4F_2 = P_{19}F_1 +$ ____.
5. Sum of the distances of each of the points P_1, P_2, P_3,... from the points F_1 and F_2 is ______.

   So, the curve obtained is an ______.

APPLICATION

This activity can be used to explain the property of an ellipse , i.e., the sum of the distances of any point on the ellipse from its two focie is constant and is equal to length of major axis.

NOTE

Construct another ellipse by taking different length of the string and also by changing the distance between F_1 and F_2.

Activity 27

OBJECTIVE

To explain the concept of octants by three mutually perpendicular planes in space.

MATERIAL REQUIRED

A piece of plywood, saw, wires, rulers wooden-board, coloured papers, scissors, cutter, thin sheet of wood, wires.

METHOD OF CONSTRUCTION

1. Cut out three square sheets each of size 30 cm × 30 cm from a piece of plywood and paste chart paper of different colours on both sides of sheets.
2. Fix two sheets in such a way that they intersect orthogonally in the middle of each other (see Fig. 27)
3. Cut the third sheet into two equal rectangles.

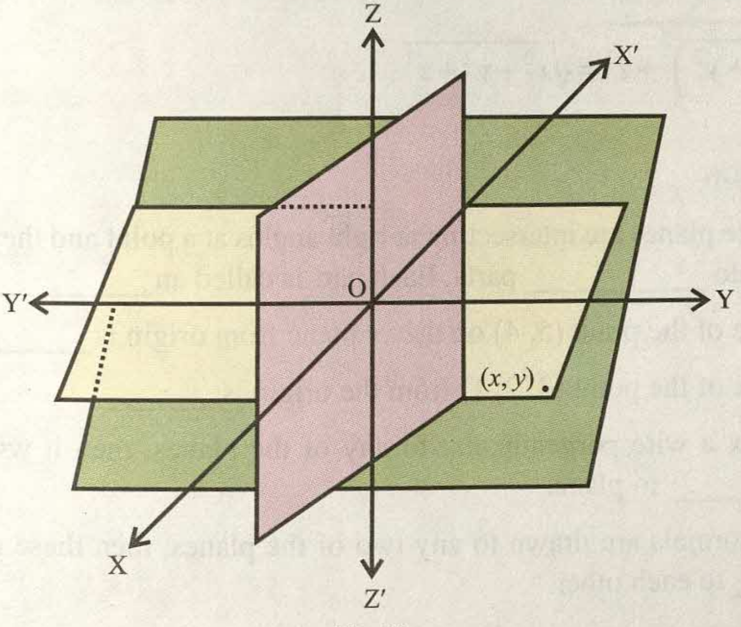


Fig. 27

4. Insert one rectangle from one side in the middle cutting the two orthogonally, and the other rectangle from the other side (see Fig. 27). The space is divided into eight parts by these three sheets. Each part is referred to as an octant.
5. Fix the model on a wooden board.
6. In one of the octants, fix rulers to represent x-axis, y-axis and z-axis. Extend each of the axis piercing to other sides to represent XX′, YY′ and ZZ′. Mark the point of intersection of XX′, YY′ and ZZ′ as origin O.

DEMONSTRATION

1. Fix a rod perpendicular to xy-plane at a point P (x, y) and parallel to z-axis.
2. Fix a wire joining the origin to the upper tip P′ (x, y, z) of this perpendicular rod.
3. The distance of point P on xy-plane with coordinates (x, y) from the origin is $\sqrt{x^2+y^2}$.
4. The distance of P′ with coordinates (x, y, z) in space from the origin is $\sqrt{\left(\sqrt{x^2+y^2}\right)^2+z^2}=\sqrt{x^2+y^2+z^2}$.

OBSERVATION

1. The three planes are intersecting at right angles at a point and they divide the space into __________ parts. Each part is called an __________.
2. Distance of the point (5, 4) on the xy plane from origin is _______.
3. Distance of the point (3, 2, 1) from the origin is _______.
4. If we fix a wire perpendicular to any of the planes, then it will represent __________ to plane.
5. If two normals are drawn to any two of the planes, then these normals are _______ to each other.

APPLICATION

1. Model can be used to visualise the position and coordinates of a point in space.
2. Model can be used to explain the distance of the origin from a point in the plane or in the space.
3. Model can also be used to explain the concept of a normal to a plane.

Activity 28

OBJECTIVE

To find analytically $\lim_{x \to c} f(x) = \frac{x^2 - c^2}{x - c}$

MATERIAL REQUIRED

Pencil, white paper, calculator.

METHOD OF CONSTRUCTION

1. Consider the function f given by $f(x) = \frac{x^2 - 9}{x - 3}$
2. In this case $c = 3$ and the function is not defined at $x = 3$.

DEMONSTRATION

1. Take some values of c less than $c = 3$ and some other values of c more than $c = 3$.
2. In both cases, the values to be taken have to be very close to $c = 3$.
3. Calculate the corresponding values of f at each of the values of c taken close to $c = 3$.

DEMONSTRATION : TABLE 1

1. Write the values of $f(x)$ in the following tables:

Table 1

x	2.9	2.99	2.999	2.9999	2.99999	2.999999
$f(x)$	5.9	5.99	5.999	5.9999	5.99999	5.999999

Table 2

x	3.1	3.01	3.001	3.0001	3.00001	3.000001
$f(x)$	6.1	6.01	6.001	6.0001	6.00001	6.000001

OBSERVATION

1. Values of $f(x)$ as $x \to 3$ from the left, as in Table 1 are coming closer and closer to _____.
2. Values of $f(x)$ as $x \to 3$ from the right, as in Table 2 are coming closer and closer to _____ from tables (2) and (3), $\lim\limits_{x \to 3} f(x) = \dfrac{x^2 - 9}{x - 3} =$ _____.

APPLICATION

This activity can be used to demonstrate the concept of a limit $\lim\limits_{x \to c} f(x)$ when $f(x)$ is not defined at $x = c$.

Activity 29

OBJECTIVE

Verification of the geometrical significance of derivative.

MATERIAL REQUIRED

Graph sheets, adhesive, hardboard, trigonometric tables, geometry box, wires.

METHOD OF CONSTRUCTION

1. Paste three graph sheets on a hardboard and draw two mutually perpendicular lines representing x-axis and y-axis on each of them.
2. Sketch the graph of the curve (circle) $x^2 + y^2 = 25$ on one sheet.
3. On the other two sheets sketch the graphs of $(x-3)^2 + y^2 = 25$ and the curve $xy = 4$ (rectangular hyperbola).

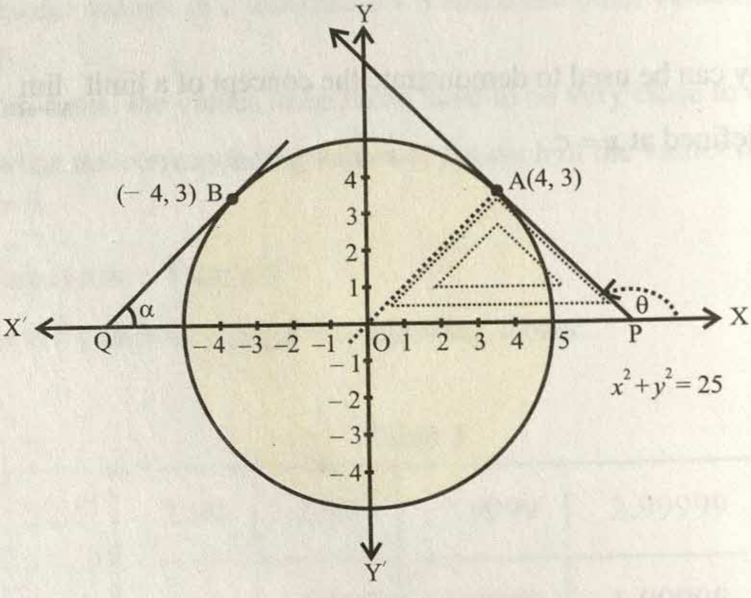


Fig. 29.1

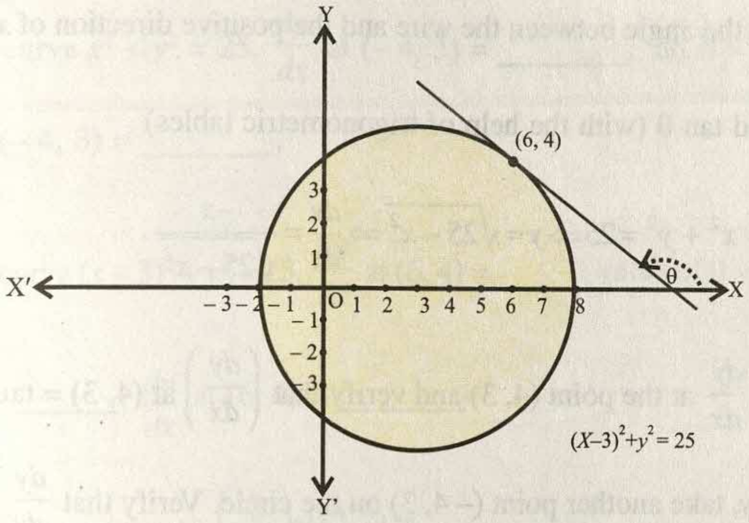


Fig 29.2

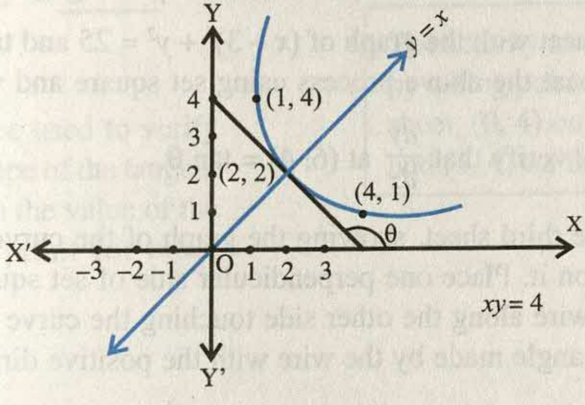


Fig. 29.3

DEMONSTRATION

1. Take first sheet on which, the graph of the circle $x^2 + y^2 = 25$ has been drawn (see Fig.29.1.
2. Take a point A (4, 3) on the circle.
3. With the help of a set square, place a wire in the direction OA and other perpendicular to OA at the point A to meet x-axis at a point (say P).

4. Measure the angle between the wire and the positive direction of x-axis at P (say θ).
5. Then find $\tan\theta$ (with the help of trigonometric tables)

$$\text{Now, } x^2+y^2=25 \Rightarrow y=\sqrt{25-x^2} \Rightarrow \frac{dy}{dx}=\frac{-x}{\sqrt{25-x^2}}.$$

Find $\frac{dy}{dx}$ at the point (4, 3) and verify that $\left(\frac{dy}{dx}\right)$ at (4, 3) = $\tan\theta$.

6. Similarly, take another point (– 4, 3) on the circle. Verify that $\frac{dy}{dx}$ at (– 4, 3) = $\tan\alpha$ where α is the angle made by the tangent to the circle at the point (– 4, 3) with the positive direction of x-axis. (see Fig. 29.1).
7. Take other sheet with the graph of $(x-3)^2+y^2=25$ and take the point (6,4) on it and repeat the above process using set square and wires as shown in Fig. 29.2, i.e. verify that $\frac{dy}{dx}$ at (6, 4) = $\tan\theta$.
8. Now take the third sheet, showing the graph of the curve $xy = 4$. Take the point (2, 2) on it. Place one perpendicular side of set square along the line $y = x$ and a wire along the other side touching the curve at the point (2, 2) and find the angle made by the wire with the positive direction of x-axis as shown in Fig. 29.3. Let it be θ. Verify that $\frac{dy}{dx}$ at (2, 2) = $\tan\theta$.

OBSERVATION

1. For the curve $x^2+y^2=25$, $\frac{dy}{dx}$ at the point (3, 4) = ________. Value of θ = _____ $\tan\theta$ = ________ $\frac{dy}{dx}$ at (3, 4) = ________.

2. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at (– 4, 3) = ________, tan α = ________

, $\frac{dy}{dx}$ at (– 4, 3) = ________.

3. For the curve $(x-3)^2 + y^2 = 25$, $\frac{dy}{dx}$ at (6, 4) = _____, value of θ = ________,

tan θ = ______, $\frac{dy}{dx}$ at (6, 4) = ________.

4. For the curve $xy = 4$, $\left(\frac{dy}{dx}\right)$at (2, 2) = ______,

θ = ____, tan θ = ______.

NOTE

The activity may be repeated by taking point (4, 3) on first sheet, (0, 4) on second sheet and (1, 4) on the third sheet.

APPLICATION

Same activity can be used to verify the result that the slope of the tangent at a point is equal to the value of the derivative at that point for other curves.

Activity 30

OBJECTIVE

To obtain truth values of compound statements of the type $p \vee q$ by using switch connections in parallel.

MATERIAL REQUIRED

Switches, electric wires, battery and lamp/bulb.

METHOD OF CONSTRUCTION

1. Connect switches S_1 and S_2 in parallel (See Fig. 30).
2. Connect battery and lamp so as to complete the circuit as shown in the figure.

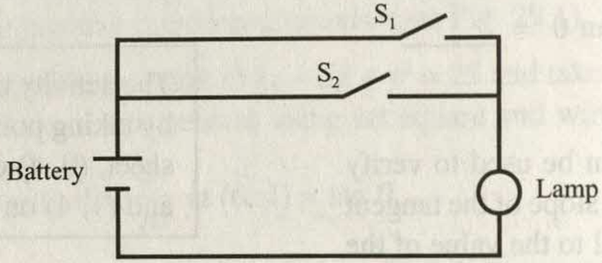


Fig. 30

DEMONSTRATION

1. The lamp will glow if atleast one of switches S_1, S_2 is on. This gives the following results:

Switch S_1	Switch S_2	Status of lamp
on	off	glow
off	on	glow
off	off	not glow
on	on	glow

Let p and q represent the statements as follows:

p : S_1 is on, truth value of p is T.

$\sim p$: S_1 is off, truth value of p is F.

q : S_2 is on, truth value of q is T.

$\sim q$: S_2 is off, truth value of q is F.

When the lamp glows, truth value of $p \vee q$ is T. When the lamp does not glow, truth value of $p \vee q$ is F. Thus, from the circuit, the following table gives the truth value of $p \vee q$:

p	q	$p \vee q$
T	F	T
F	T	T
F	F	F
T	T	T

OBSERVATION

1. If S_1 is on, truth value of p is _______________.
   If S_1 is off, truth value of p is _______________.
   If S_2 is on, truth value of q is _______________.
   If S_2 is off, truth value of q is _______________.
2. If S_1 is on, S_2 is off, truth value of $p \vee q$ is ________.
   If S_1 is on, S_2 is on, truth value of $p \vee q$ is ________.
   If S_1 is off, S_2 is off, truth value of $p \vee q$ is ________.
   If S_1 is off, S_2 is on, truth value of $p \vee q$ is ________.
   If S_1 is ______, S_2 is _____, truth value of $p \vee q$ is T.

APPLICATION

This activity helps in understanding truth values of the statements $p \vee q$ in different cases of the statements p and q.

Activity 31

OBJECTIVE

To obtain truth values of compound statements of the type $p \wedge q$ by using switch connections in series.

MATERIAL REQUIRED

Switches, electric wires, battery and lamp/bulb.

METHOD OF CONSTRUCTION

1. Connect switches S_1 and S_2 in series (See Fig. 31)
2. Connect battery and lamp so as to complete the circuit as shown in the figure.

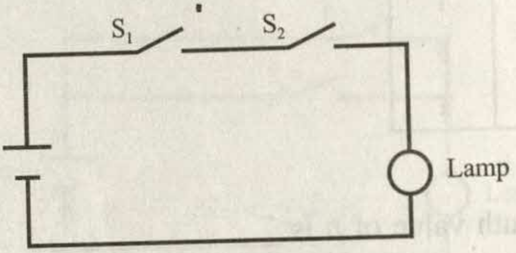


Fig. 31

DEMONSTRATION

1. The lamp will glow if both the switches S_1 and S_2 together are on. This gives the following results :

Switch S_1	Switch S_2	Status of lamp
on	off	not glow
on	on	glow
off	on	not glow
off	off	not glow

Let p and q represent the statements as follows :

p : S_1 is on, truth value of p is T.
$\sim p$: S_1 is off, truth value of p is F.
q : S_2 is on, truth value of q is T.
$\sim q$: S_2 is off, truth value of q is F.

When the lamp glows, truth value of $p \wedge q$ is T. When the lamp does not glow, truth value of $p \wedge q$ is F. Thus, from the circuit, the following table gives the truth values of $p \wedge q$.

p	q	$p \wedge q$
T	T	T
F	T	F
T	F	F
F	F	F

OBSERVATION

1. If S_1 is on, truth value of p is ______________.
   If S_1 is off, truth value of p is ______________.
   If S_2 is on, truth value of q is ______________.
   If S_2 is off, truth value of q is ______________.
2. If S_1 is on, S_2 is off, truth value of $p \wedge q$ is ________.
   If S_1 is on, S_2 is on, truth value of $p \wedge q$ is ________.
   If S_1 is off, S_2 is off, truth value of $p \wedge q$ is ________.
   If S_1 is off, S_2 is on, truth value of $p \wedge q$ is ________.
   If S_1 is ______, S_2 is ____, truth value of $p \wedge q$ is T.

APPLICATION

This activity may help the students in understandig truth values of the statements $p \wedge q$ in different cases of the statements p and q.

Activity 32

OBJECTIVE

To write the sample space, when a die is rolled once, twice --------

MATERIAL REQUIRED

A die, paper, pencil/pen, plastic discs, marked with 1, 2, 3, 4, 5 or 6.

METHOD OF CONSTRUCTION

1. Throw a die once. The number on its top will be 1, 2, 3, 4, 5 or 6.
2. Make a tree diagram showing its six branches with number 1,2, 3, 4, 5 or 6 (See Fig. 32.1)
3. Write the sample space of these outcomes.
4. Throw a die twice. It can fall in any of the 36 ways as shown in Fig. 32.2 by the tree diagram. Write the sample space of these outcomes.

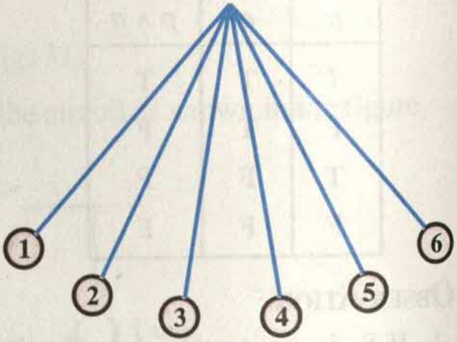


Fig 32.1

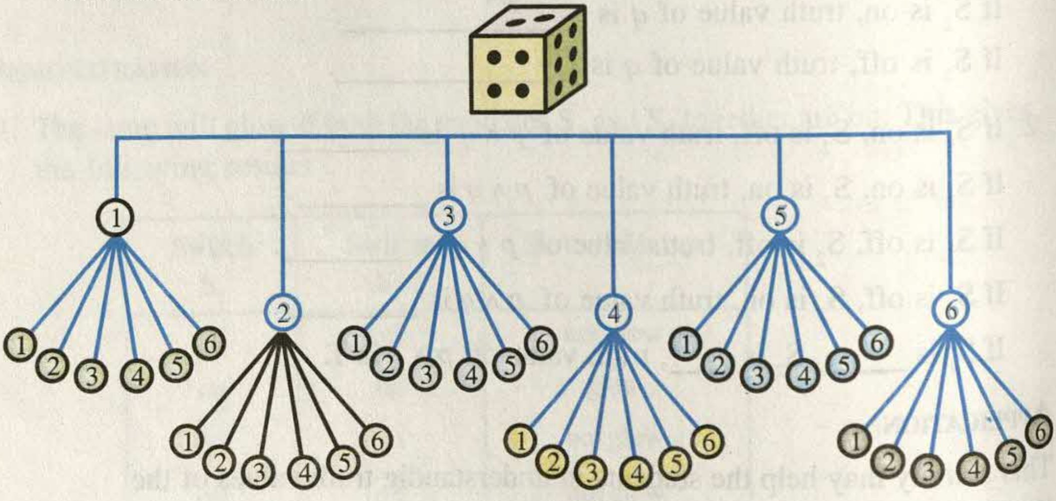


Fig 32.2

5. Repeat the experiment by throwing a die 3 times, and write the sample space of the outcomes using a tree diagram.

DEMONSTRATION

1. If a die is thrown once, the sample space is

   $S = \{1, 2, 3, 4, 5, 6\}$. Number of elements in $S = 6 = 6^1$

2. If a die is thrown twice, the sample space is

$$\text{Sample space } S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

The number of elements in $S = 36 = 6^2$ and so on.

OBSERVATION

Number of elements in sample space when a die is thrown

Once = __________, Thrice = ________, Four times = _________

APPLICATION

Sample space of an experiment is useful in determining the probabilities of different events associated with the sample space.

Activity 33

OBJECTIVE

To write the sample space, when a coin is tossed once, two times, three times, four times.

MATERIAL REQUIRED

One rupee coin, paper pencil/pen, plastic circular discs, marked with Head (H) and Tail (T).

METHOD OF CONSTRUCTION

1. Toss a coin once. It can have two outcomes – Head or Tail.
2. Make a tree diagram showing the two branches of a tree - with H (Head) on one branch and T (Tail) on the other (see Fig. 33.1).
3. Write its sample space.
4. Toss a coin twice. It can have four outcomes (see Fig. 33.2)
5. Repeat the experiment with tossing the coin three times, four times, ..., *n* and write their sample spaces, if possible. (see Fig. 33.3 and 33.4).

DEMONSTRATION

1. If a coin is tossed once, the sample space is

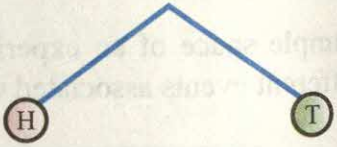


Fig. 33.1

$$S = \{H, T\}$$

Number of elements in $S = 2 = 2^1$

2. When a coin is tossed twice, the sample space is

$$S = \{HH, HT, TH, TT\}$$

Number of elements in $S = 4 = 2^2$

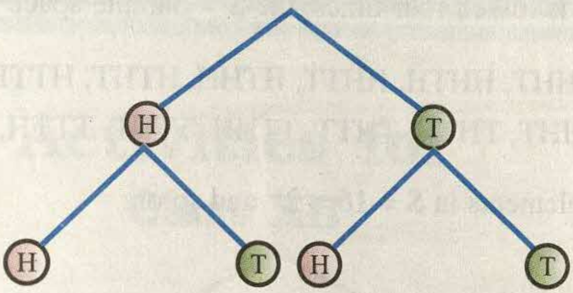


Fig. 33.2

3. When a coin is tossed three times, the sample space is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Number of elements in $S = 8 = 2^3$

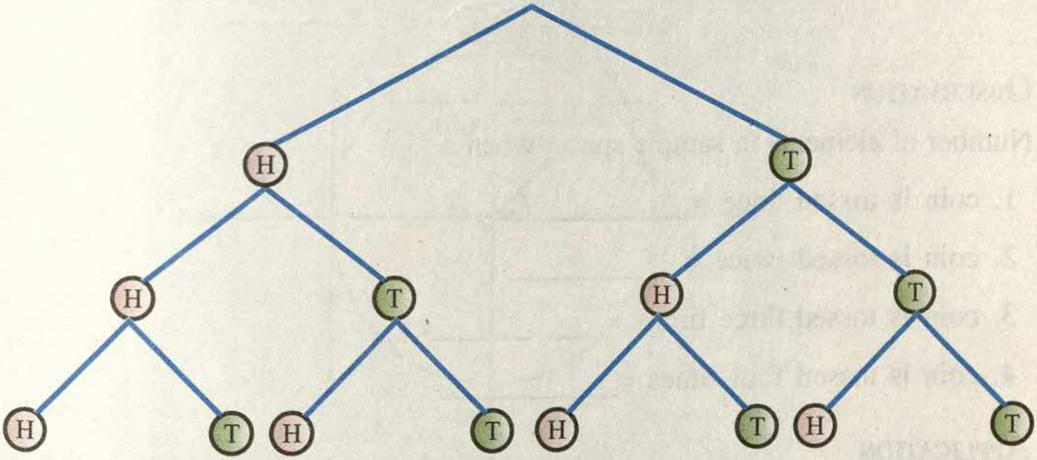


Fig. 33.3

4. When a coin is tossed four times, the S = Sample space is

$$\left\{\begin{matrix}\text{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT,} \\ \text{THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}\end{matrix}\right\}$$

Number of elements in S = 16 = 2^4 and so on.

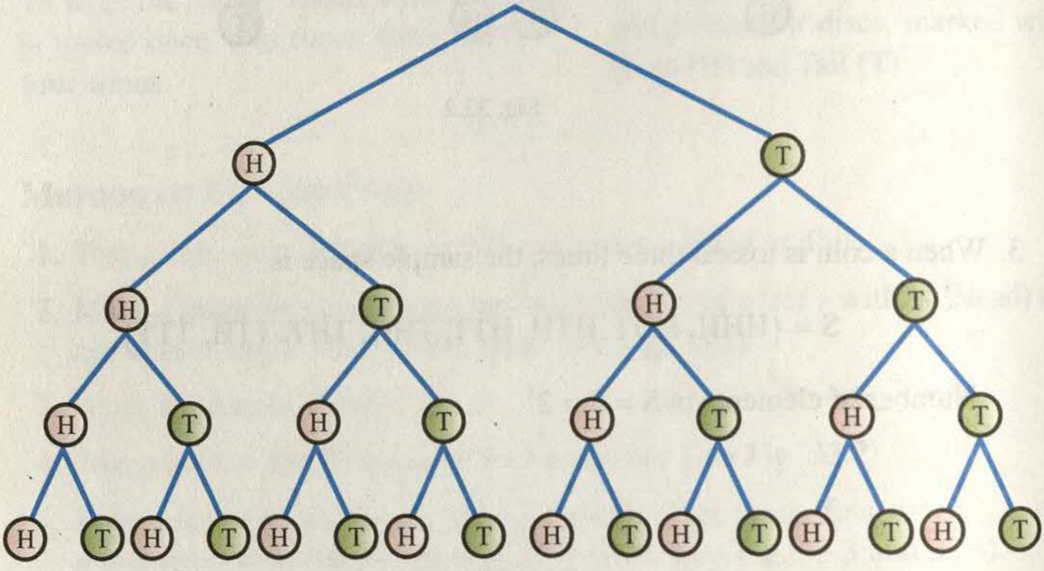


Fig. 33.4

OBSERVATION

Number of elements in sample space, when a

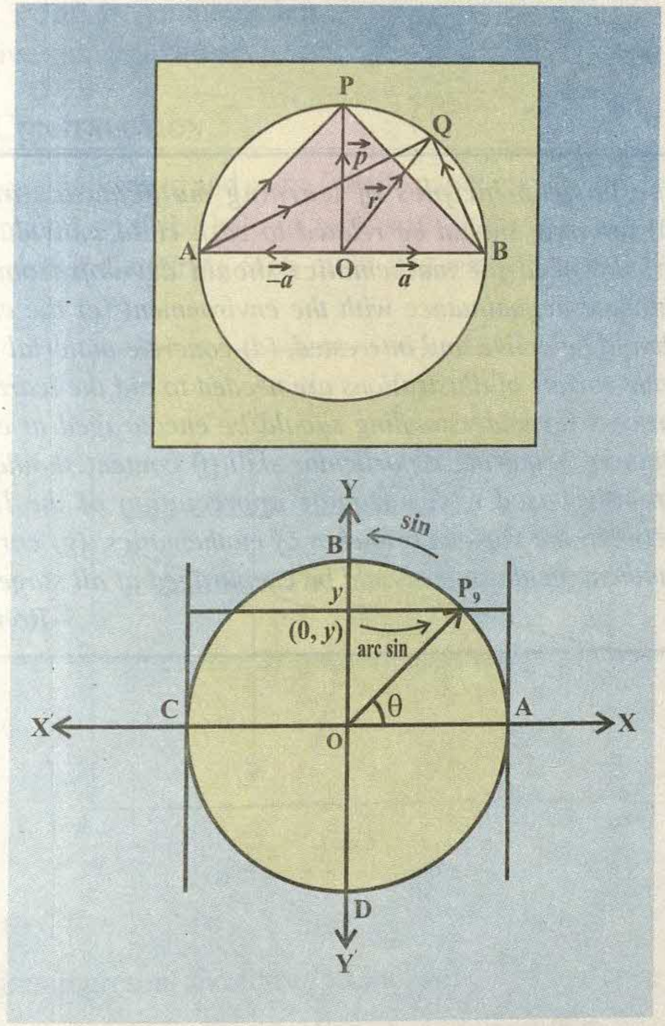
1. coin is tossed once = __________.
2. coin is tossed twice = __________.
3. coin is tossed three times = __________.
4. coin is tossed four times = __________.

APPLICATION

Sample space of an experiment is useful in determining the probabilities of different events associated with the sample space.

Activities for

Class XII



The basic principles of learning mathematics are : (a) learning should be related to each child individually (b) the need for mathematics should develop from an intimate acquaintance with the environment (c) the child should be active and interested, (d) concrete material and wide variety of illustrations are needed to aid the learning process (e) understanding should be encouraged at each stage of acquiring a particular skill (f) content should be broadly based with adequate appreciation of the links between the various branches of mathematics, (g) correct mathematical usage should be encouraged at all stages.

–Ronwill

Activity 1

OBJECTIVE

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \perp m\}$ is symmetric but neither reflexive nor transitive.

MATERIAL REQUIRED

A piece of plywood, some pieces of wires (8), nails, white paper, glue etc.

METHOD OF CONSTRUCTION

Take a piece of plywood and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig.1.

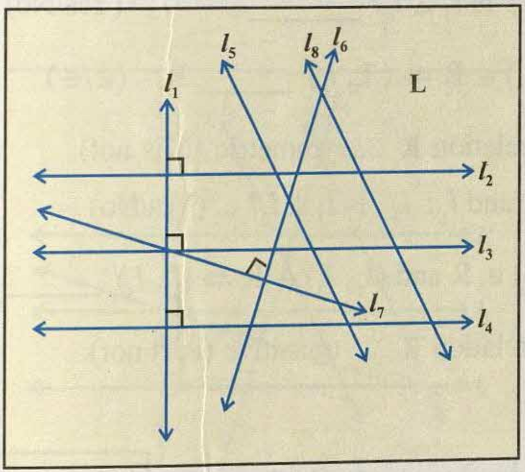


Fig. 1

DEMONSTRATION

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4. [see Fig. 1]

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in R$

OBSERVATION

1. In Fig. 1, no line is perpendicular to itself, so the relation $R = \{(l, m) : l \perp m\}$ ______ reflexive (is/is not).

2. In Fig. 1, $l_1 \perp l_2$. Is $l_2 \perp l_1$? ______ (Yes/No)

   $\therefore$ $(l_1, l_2) \in R \Rightarrow (l_2, l_1)$ ______ R $(\notin / \in)$

   Similarly, $l_3 \perp l_1$. Is $l_1 \perp l_3$? ______ (Yes/No)

   $\therefore$ $(l_3, l_1) \in R \Rightarrow (l_1, l_3)$ ______ R $(\notin / \in)$

   Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? ______ (Yes/No)

   $\therefore$ $(l_6, l_7) \in R \Rightarrow (l_7, l_6)$ ______ R $(\notin / \in)$

   $\therefore$ The relation R .... symmetric (is/is not)

3. In Fig. 1, $l_2 \perp l_1$ and $l_1 \perp l_3$. Is $l_2 \perp l_3$? ... (Yes/No)

   i.e., $(l_2, l_1) \in R$ and $(l_1, l_3) \in R \Rightarrow (l_2, l_3)$ ______ R $(\notin / \in)$

   $\therefore$ The relation R .... transitive (is/is not).

APPLICATION

This activity can be used to check whether a given relation is an equivalence relation or not.

NOTE

1. In this case, the relation is not an equivalence relation.
2. The activity can be repeated by taking some more wire in different positions.

Activity 2

OBJECTIVE

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \| m\}$ is an equivalence relation.

MATERIAL REQUIRED

A piece of plywood, some pieces of wire (8), plywood, nails, white paper, glue.

METHOD OF CONSTRUCTION

Take a piece of plywood of convenient size and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig. 2.

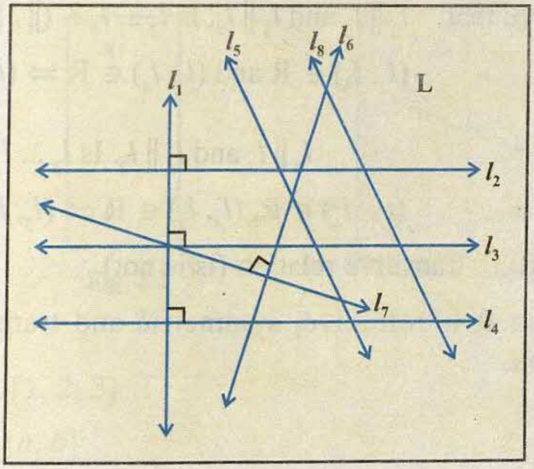


Fig. 2

DEMONSTRATION

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 (see Fig. 2).

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_2, l_3), (l_3, l_4), (l_5, l_8), \in$ R

OBSERVATION

1. In Fig. 2, every line is parallel to itself. So the relation R = $\{(l, m) : l \parallel m\}$ .... reflexive relation (is/is not)

2. In Fig. 2, observe that $l_2 \parallel l_3$. Is l_3 ... l_2? $(\nparallel / \parallel)$

So, $(l_2, l_3) \in \text{R} \Rightarrow (l_3, l_2) \ldots \text{R} \ (\notin / \in)$

Similarly, $l_3 \parallel l_4$. Is $l_4 \ldots l_3$? $(\nparallel / \parallel)$

So, $(l_3, l_4) \in \text{R} \Rightarrow (l_4, l_3) \ldots \text{R} \ (\notin / \in)$

and $(l_5, l_8) \in \text{R} \Rightarrow (l_8, l_5) \ldots \text{R} \ (\notin / \in)$

The relation R ... symmetric relation (is/is not)

3. In Fig. 2, observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is $l_2 \ldots l_4$? $(\parallel / \parallel)$

So, $(l_2, l_3) \in \text{R}$ and $(l_3, l_4) \in \text{R} \Rightarrow (l_2, l_4) \ldots \text{R} \ (\in / \notin)$

Similarly, $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is $l_3 \ldots l_2$? $(\nparallel / \parallel)$

So, $(l_3, l_4) \in \text{R}, (l_4, l_2) \in \text{R} \Rightarrow (l_3, l_2) \ldots \text{R} \ (\in , \notin)$

Thus, the relation R ... transitive relation (is/is not)

Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

APPLICATION

This activity is useful in understanding the concept of an equivalence relation.

NOTE

This activity can be repeated by taking some more wires in different positions.

Activity 3

OBJECTIVE

To demonstrate a function which is not one-one but is onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix three nails on it as shown in the Fig.3.1. Name the nails on the strip as 1, 2 and 3.
2. Paste another strip on the right hand side of the cardboard and fix two nails in the plastic strip as shown in Fig.3.2. Name the nails on the strip as *a* and *b*.
3. Join nails on the left strip to the nails on the right strip as shown in Fig. 3.3.

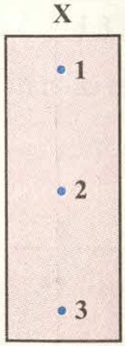


Fig. 3.1

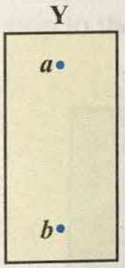


Fig. 3.2

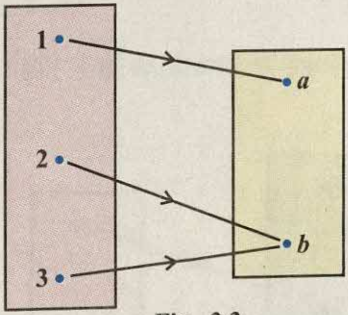


Fig. 3.3

DEMONSTRATION

1. Take the set X = {1, 2, 3}
2. Take the set Y = {*a*, *b*}
3. Join (correspondence) elements of X to the elements of Y as shown in Fig. 3.3

OBSERVATION

1. The image of the element 1 of X in Y is ________.

   The image of the element 2 of X in Y is ________.

The image of the element 3 of X in Y is ________.

So, Fig. 3.3 represents a ________ .

2. Every element in X has a ________ image in Y. So, the function is ________(one-one/not one-one).
3. The pre-image of each element of Y in X ________ (exists/does not exist). So, the function is ________ (onto/not onto).

APPLICATION

This activity can be used to demonstrate the concept of one-one and onto function.

NOTE

Demonstrate the same activity by changing the number of the elements of the sets X and Y.

Activity 4

OBJECTIVE

To demonstrate a function which is one-one but not onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix two nails in it as shown in the Fig. 4.1. Name the nails as *a* and *b*.
2. Paste another strip on the right hand side of the cardboard and fix three nails on it as shown in the Fig. 4.2. Name the nails on the right strip as 1, 2 and 3.
3. Join nails on the left strip to the nails on the right strip as shown in the Fig. 4.3.

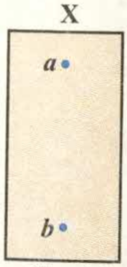


Fig. 4.1

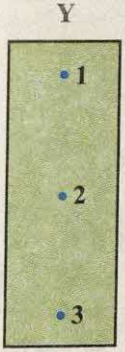


Fig. 4.2

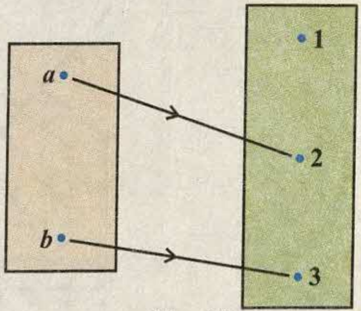


Fig. 4.3

DEMONSTRATION

1. Take the set X = {*a*, *b*}
2. Take the set Y = {1, 2, 3}.
3. Join elements of X to the elements of Y as shown in Fig. 4.3.

OBSERVATION

1. The image of the element a of X in Y is ____________.

   The image of the element b of X in Y is ____________.

   So, the Fig. 4.3 represents a ______________.

2. Every element in X has a _________ image in Y. So, the function is ____________ (one-one/not one-one).

3. The pre-image of the element 1 of Y in X _________ (exists/does not exist). So, the function is _________ (onto/not onto).

   Thus, Fig. 4.3 represents a function which is ________ but not onto.

APPLICATION

This activity can be used to demonstrate the concept of one-one but not onto function.

Activity 5

OBJECTIVE

To draw the graph of $\sin^{-1} x$, using the graph of sin x and demonstrate the concept of mirror reflection (about the line $y = x$).

MATERIAL REQUIRED

Cardboard, white chart paper, ruler, coloured pens, adhesive, pencil, eraser, cutter, nails and thin wires.

METHOD OF CONSTRUCTION

1. Take a cardboard of suitable dimensions, say, 30 cm × 30 cm.
2. On the cardboard, paste a white chart paper of size 25 cm × 25 cm (say).
3. On the paper, draw two lines, perpendicular to each other and name them X′OX and YOY′ as rectangular axes [see Fig. 5].

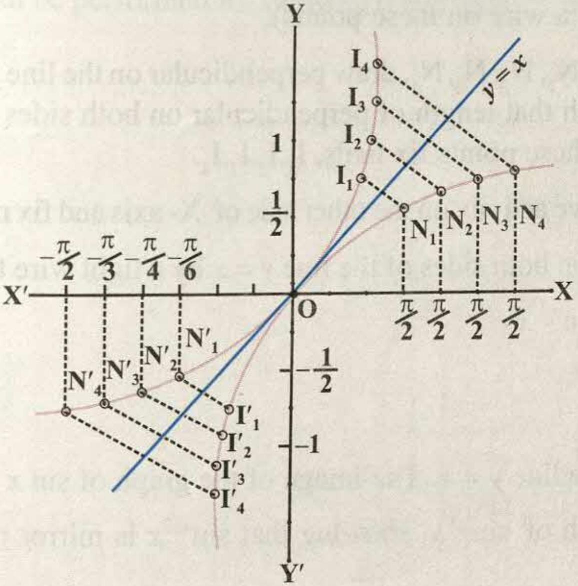


Fig. 5

4. Graduate the axes approximately as shown in Fig. 5.1 by taking unit on X-axis = 1.25 times the unit of Y-axis.

5. Mark approximately the points

   $\left(\frac{\pi}{6}, \sin\frac{\pi}{6}\right), \left(\frac{\pi}{4}, \sin\frac{\pi}{4}\right), \ldots, \left(\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$ in the coordinate plane and at each point fix a nail.

6. Repeat the above process on the other side of the x-axis, marking the points

   $\left(\frac{-\pi}{6}, \sin\frac{-\pi}{6}\right), \left(\frac{-\pi}{4}, \sin\frac{-\pi}{4}\right), \ldots, \left(\frac{-\pi}{2}, \sin\frac{-\pi}{2}\right)$ approximately and fix nails on these points as N_1', N_2', N_3', N_4'. Also fix a nail at O.

7. Join the nails with the help of a tight wire on both sides of x-axis to get the graph of $\sin x$ from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.

8. Draw the graph of the line $y = x$ (by plotting the points (1,1), (2, 2), (3, 3), ... etc. and fixing a wire on these points).

9. From the nails N_1, N_2, N_3, N_4, draw perpendicular on the line $y = x$ and produce these lines such that length of perpendicular on both sides of the line $y = x$ are equal. At these points fix nails, I_1, I_2, I_3, I_4.

10. Repeat the above activity on the other side of X- axis and fix nails at I_1', I_2', I_3', I_4'.

11. Join the nails on both sides of the line $y = x$ by a tight wire that will show the graph of $y = \sin^{-1} x$.

DEMONSTRATION

Put a mirror on the line $y = x$. The image of the graph of $\sin x$ in the mirror will represent the graph of $\sin^{-1} x$ showing that $\sin^{-1} x$ is mirror reflection of $\sin x$ and vice versa.

OBSERVATION

The image of point N_1 in the mirror (the line $y = x$) is ________.

The image of point N_2 in the mirror (the line $y = x$) is ________.

The image of point N_3 in the mirror (the line $y = x$) is ________.

The image of point N_4 in the mirror (the line $y = x$) is ________.

The image of point N'_1 in the mirror (the line $y = x$) is ________.

The image point of N'_2 in the mirror (the line $y = x$) is ________.

The image point of N'_3 in the mirror (the line $y = x$) is ________.

The image point of N'_4 in the mirror (the line $y = x$) is ________.

The image of the graph of six x in $y = x$ is the graph of ________, and the image of the graph of $\sin^{-1}x$ in $y = x$ is the graph of ________.

APPLICATION

Similar activity can be performed for drawing the graphs of $\cos^{-1}x$, $\tan^{-1}x$, etc.

Activity 6

OBJECTIVE

To explore the principal value of he function $\sin^{-1}x$ using a unit circle.

MATERIAL REQUIRED

Cardboard, white chart paper, rails, ruler, adhesive, steel wires and needle.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on it.
3. Through the centre of the circle, draw two perpendicular lines X′OX and YOY′ representing x-axis and y-axis, respectively as shown in Fig. 6.1.
4. Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis, respectively as shown in Fig. 6.1.
5. Fix two rails on opposite sides of the cardboard which are parallel to y-axis. Fix one steel wire between the rails such that the wire can be moved parallel to x-axis as shown in Fig. 6.2.

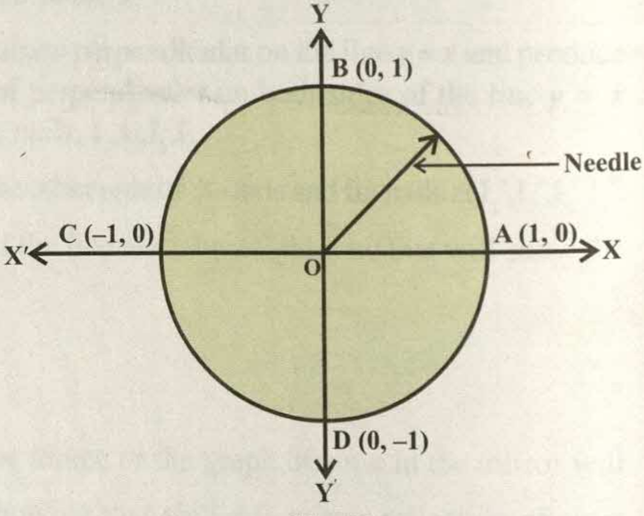


Fig. 6.1

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle Fig. 6.2.

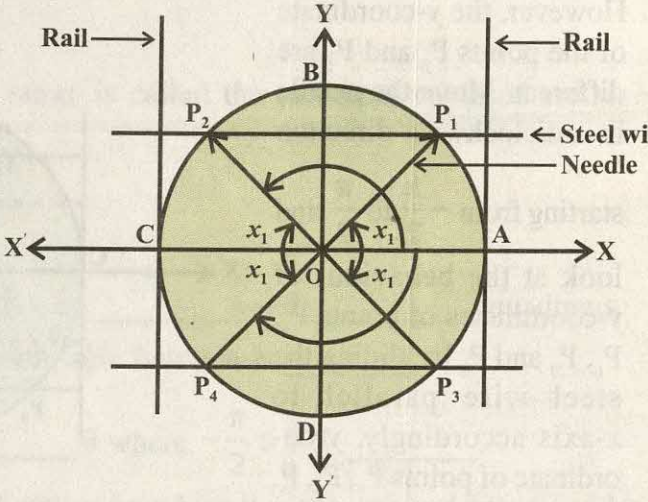


Fig. 6.2

DEMONSTRATION

1. Keep the needle at an arbitrary angle, say x_1 with the positive direction of x-axis. Measure of angle in radian is equal t the length of intercepted arc of the unit circle.

2. Slide the steel wire between the rails, parallel to x-axis such that the wi meets with free end of the needle (say P_1) (Fig. 6.2).

3. Denote the y-coordinate of the point P_1 as y_1, where y_1 is the perpendicul distance of steel wire from the x-axis of the unit circle giving $y_1 = \sin x_1$

4. Rotate the needle further anticlockwise and keep it at the angle $\pi - x_1$. Fin the value of y-coordinate of intersecting point P_2 with the help of slidin steel wire. Value of y-coordinate for the points P_1 and P_2 are same for th different value of angles, $y_1 = \sin x_1$ and $y_1 = \sin(\pi - x_1)$. This demonstrate that sine function is not one-to-one for angles considered in first and secon quadrants.

5. Keep the needle at angles $-x_1$ and $(-\pi + x_1)$, respectively. By sliding dow the steel wire parallel to x-axis, demonstrate that y-coordinate for the poin P_3 and P_4 are the same and thus sine function is not one-to-one for poin considered in 3rd and 4th quadrants as shown in Fig. 6.2.

6. However, the y-coordinate of the points P_3 and P_1 are different. Move the needle in anticlockwise direction starting from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and look at the behaviour of y-coordinates of points P_5, P_6, P_7 and P_8 by sliding the steel wire parallel to x-axis accordingly. y-coordinate of points P_5, P_6, P_7 and P_8 are different (see Fig. 6.3). Hence, sine function is one-to-one in the domian $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and its range lies between – 1 and 1.

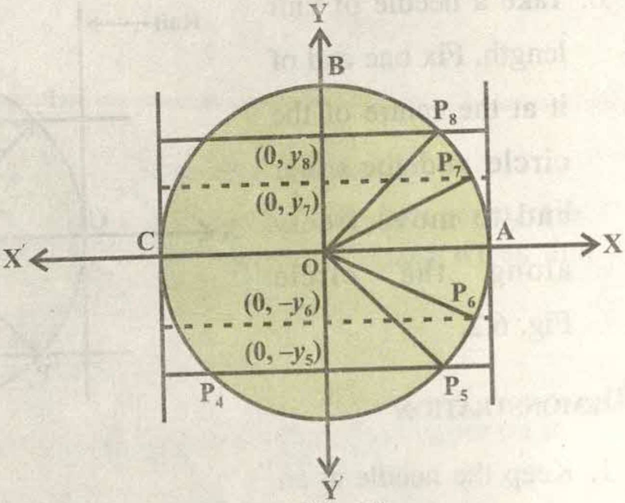


Fig. 6.3

7. Keep the needle at any arbitrary angle say θ lying in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and denote the y-coordinate of the intersecting point P_9 as y. (see Fig. 6.4). Then $y = \sin\theta$ or $\theta = \text{arc}\sin^{-1}y$) as sine function is one-one and onto in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and range [–1, 1]. So, its inverse arc sine function exist. The domain of arc sine function is [–1, 1] and

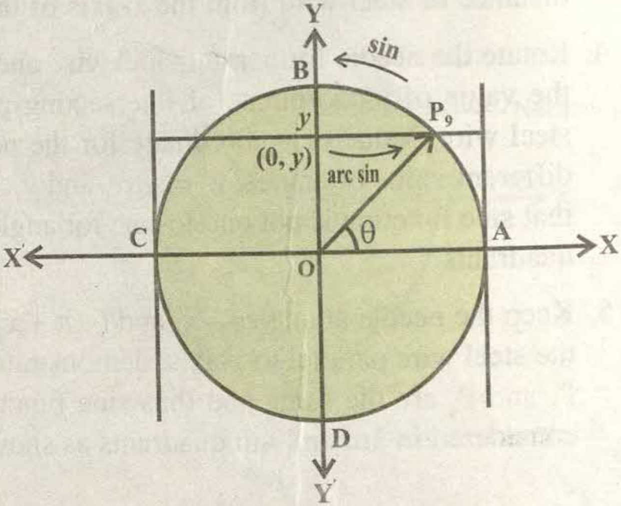


Fig. 6.4

range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This range is called the principal value of arc sine function (or $\sin^{-1}$ function).

OBSERVATION

1. sine function is non-negative in ________ and ________ quadrants.
2. For the quadrants 3rd and 4th, sine function is ________.
3. $\theta = \text{arc}\sin y \Rightarrow y =$ ________ θ where $-\frac{\pi}{2} \leq \theta \leq$ ________.
4. The other domains of sine function on which it is one-one and onto provides ________ for arc sine function.

APPLICATION

This activity can be used for finding the principal value of arc cosine function ($\cos^{-1}y$).

Activity 7

OBJECTIVE

To sketch the graphs of a^x and $\log_a x$, $a > 0, a \neq 1$ and to examine that they are mirror images of each other.

MATERIAL REQUIRED

Drawing board, geometrical instruments, drawing pins, thin wires, sketch pens, thick white paper, adhesive, pencil, eraser, a plane mirror, squared paper.

METHOD OF CONSTRUCTION

1. On the drawing board, fix a thick paper sheet of convenient size 20 cm × 20 cm (say) with adhesive.

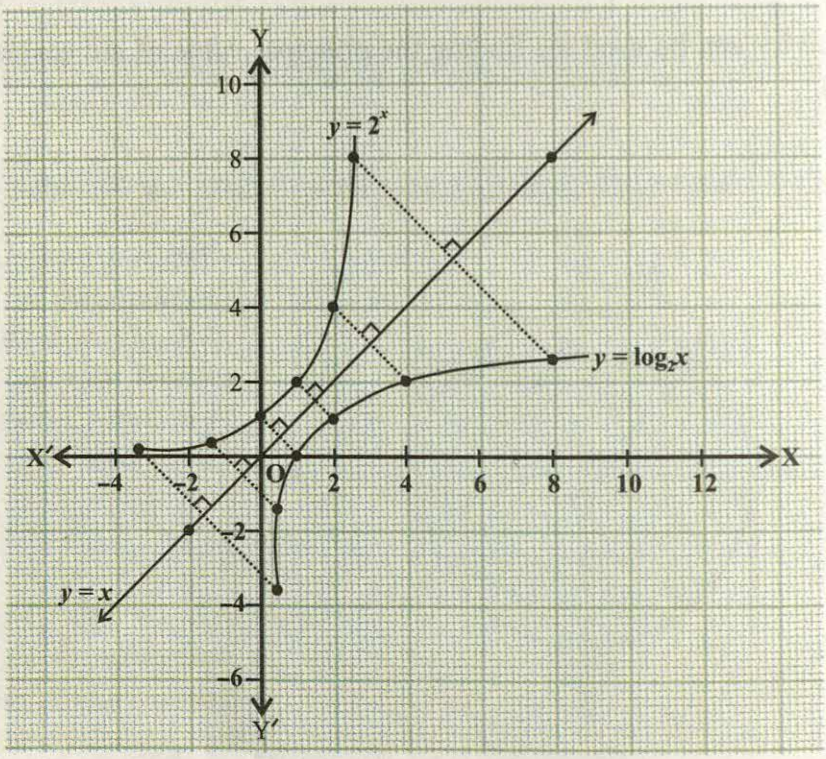


Fig. 7

2. On the sheet, take two perpendicular lines XOX′ and YOY′, depicting coordinate axes.
3. Mark graduations on the two axes as shown in the Fig. 7.
4. Find some ordered pairs satisfying $y = a^x$ and $y = \log_a x$. Plot these points corresponding to the ordered pairs and join them by free hand curves in both the cases. Fix thin wires along these curves using drawing pins.
5. Draw the graph of $y = x$, and fix a wire along the graph, using drawing pins.

DEMONSTRATION

1. For a^x, take $a = 2$ (say), and find ordered pairs satisfying it as

x	0	1	–1	2	–2	3	–3	$\frac{1}{2}$	$-\frac{1}{2}$	4
2^x	1	2	0.5	4	$\frac{1}{4}$	8	$\frac{1}{8}$	1.4	0.7	16

and plot these ordered pairs on the squared paper and fix a drawing pin at each point.

2. Join the bases of drawing pins with a thin wire. This will represent the graph of 2^x.

3. $\log_2 x = y$ gives $x = 2^y$. Some ordered pairs satisfying it are:

x	1	2	$\frac{1}{2}$	4	$\frac{1}{4}$	8	$\frac{1}{8}$
y	0	1	–1	2	–2	3	–3

Plot these ordered pairs on the squared paper (graph paper) and fix a drawing pin at each plotted point. Join the bases of the drawing pins with a thin wire. This will represent the graph of $\log_2 x$.

4. Draw the graph of line $y = x$ on the sheet.
5. Place a mirror along the wire representing $y = x$. It can be seen that the two graphs of the given functions are mirror images of each other in the line $y = x$.

OBSERVATION

1. Image of ordered pair (1, 2) on the graph of $y = 2^x$ in $y = x$ is ______. It lies on the graph of $y =$ ______.
2. Image of the point (4, 2) on the graph $y = \log_2 x$ in $y = x$ is ________ which lies on the graph of $y =$ _______.

Repeat this process for some more points lying on the two graphs.

APPLICATION

This activity is useful in understanding the concept of (exponential and logarithmic functions) which are mirror images of each other in $y = x$.

Activity 8

OBJECTIVE

To establish a relationship between common logarithm (to the base 10) and natural logarithm (to the base e) of the number x.

MATERIAL REQUIRED

Hardboard, white sheet, graph paper, pencil, scale, log tables or calculator (graphic/scientific).

METHOD OF CONSTRUCTION

1. Paste a graph paper on a white sheet and fix the sheet on the hardboard.
2. Find some ordered pairs satisfying the function $y = \log_{10} x$. Using log tables/ calculator and draw the graph of the function on the graph paper (see Fig. 8)

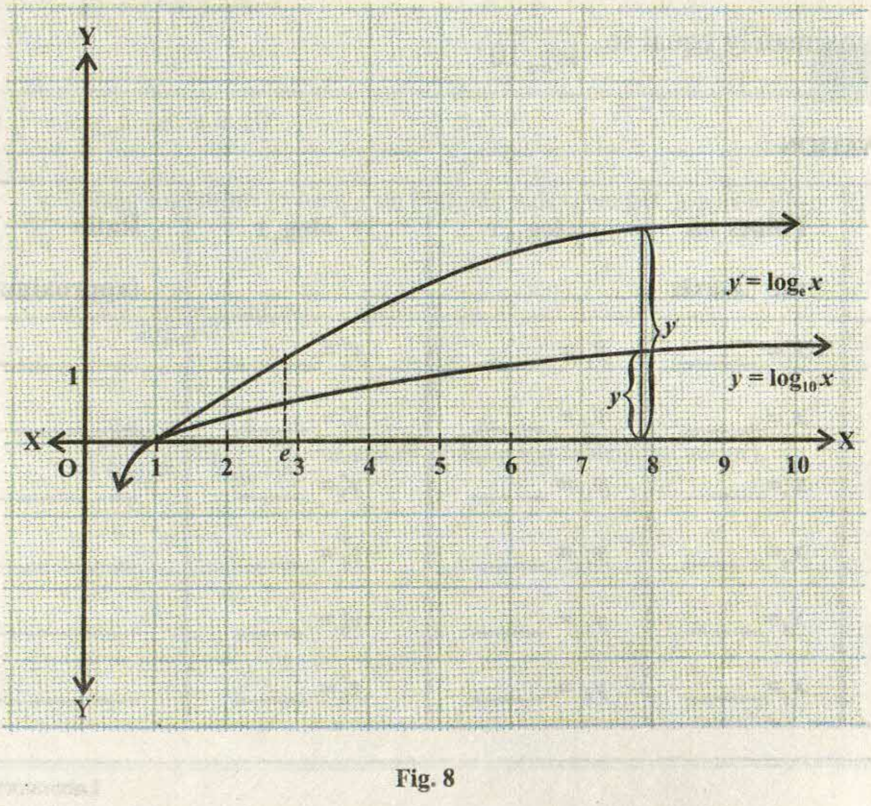


Fig. 8

3. Similarly, draw the graph of $y' = \log_e x$ on the same graph paper as shown in the figure (using log table/calculator).

DEMONSTRATION

1. Take any point on the positive direction of x-axis, and note its x-coordinate.
2. For this value of x, find the value of y-coordinates for both the graphs of $y = \log_{10} x$ and $y' = \log_e x$ by actual measurement, using a scale, and record them as y and y', respectively.
3. Find the ratio $\frac{y}{y'}$.
4. Repeat the above steps for some more points on the x-axis (with different values) and find the corresponding ratios of the ordinates as in Step 3.
5. Each of these ratios will nearly be the same and equal to 0.4, which is approximately equal to $\frac{1}{\log_e 10}$.

OBSERVATION

S.No.	Points on the x-axis	$y = \log_{10} x$	$y' = \log_e x$	Ratio $\frac{y}{y'}$ (approximate)
1.	$x_1 =$ _____	$y_1 =$ _____	$y'_1 =$ _____	_______
2.	$x_2 =$ _____	$y_2 =$ _____	$y'_2 =$ _____	_______
3.	$x_3 =$ _____	$y_3 =$ _____	$y'_3 =$ _____	_______
4.	$x_4 =$ _____	$y_4 =$ _____	$y'_4 =$ _____	_______
5.	$x_5 =$ _____	$y_5 =$ _____	$y'_5 =$ _____	_______
6.	$x_6 =$ _____	$y_6 =$ _____	$y'_6 =$ _____	_______

2. The value of $\frac{y}{y'}$ for each point x is equal to ________ approximately.

3. The observed value of $\frac{y}{y'}$ in each case is approximately equal to the value of $\frac{1}{\log_e 10}$. (Yes/No)

4. Therefore, $\log_{10} x = \frac{\qquad}{\log_e 10}$.

APPLICATION

This activity is useful in converting log of a number in one given base to log of that number in another base.

NOTE

Let, $y = \log_{10} x$, i.e., $x = 10^y$.

Taking logarithm to base e on both the sides, we get $\log_e x = y \log_e 10$

$$\text{or } y = \frac{1}{\log_e 10}(\log_e x)$$

$$\Rightarrow \frac{\log_{10} x}{\log_e x} = \frac{1}{\log_e 10} = 0.434294 \text{ (using log tables/calculator).}$$

Activity 9

OBJECTIVE

To find analytically the limit of a function $f(x)$ at $x = c$ and also to check the continuity of the function at that point.

MATERIAL REQUIRED

Paper, pencil, calculator.

METHOD OF CONSTRUCTION

1. Consider the function given by $f(x) = \begin{cases} \dfrac{x^2-16}{x-4}, & x \neq 4 \\ 10, & x = 4 \end{cases}$
2. Take some points on the left and some points on the right side of c (= 4) which are very near to c.
3. Find the corresponding values of $f(x)$ for each of the points considered in step 2 above.
4. Record the values of points on the left and right side of c as x and the corresponding values of $f(x)$ in a form of a table.

DEMONSTRATION

1. The values of x and $f(x)$ are recorded as follows:

Table 1 : For points on the left of c (= 4).

x	3.9	3.99	3.999	3.9999	3.99999	3.999999	3.9999999
$f(x)$	7.9	7.99	7.999	7.9999	7.99999	7.999999	7.9999999

2. **Table 2**: For points on the right of c (= 4).

x	4.1	4.01	4.001	4.0001	4.00001	4.000001	4.0000001
$f(x)$	8.1	8.01	8.001	8.0001	8.00001	8.000001	8.0000001

OBSERVATION

1. The value of $f(x)$ is approaching to ________, as $x \to 4$ from the left.
2. The value of $f(x)$ is approaching to ________, as $x \to 4$ from the right.
3. So, $\lim\limits_{x \to 4^-} f(x) =$ ________ and $\lim\limits_{x \to 4^+} f(x) =$ ________.
4. Therefore, $\lim\limits_{x \to 4} f(x) =$ ________ , $f(4) =$ ________.
5. Is $\lim\limits_{x \to 4} f(x) = f(4)$ ________ ? (Yes/No)
6. Since $f(c) \neq \lim\limits_{x \to c} f(x)$, so, the function is ________ at $x = 4$ (continuous/ not continuous).

APPLICATION

This activity is useful in understanding the concept of limit and continuity of a function at a point.

Activity 10

OBJECTIVE

To verify that for a function f to be continuous at given point x_0,

$\Delta y = |f(x_0 + \Delta x) - f(x_0)|$ is

arbitrarily small provided. Δx is sufficiently small.

MATERIAL REQUIRED

Hardboard, white sheets, pencil, scale, calculator, adhesive.

METHOD OF CONSTRUCTION

1. Paste a white sheet on the hardboard.
2. Draw the curve of the given continuous function as represented in the Fig. 10.
3. Take any point A $(x_0, 0)$ on the positive side of x-axis and corresponding to this point, mark the point P (x_0, y_0) on the curve.

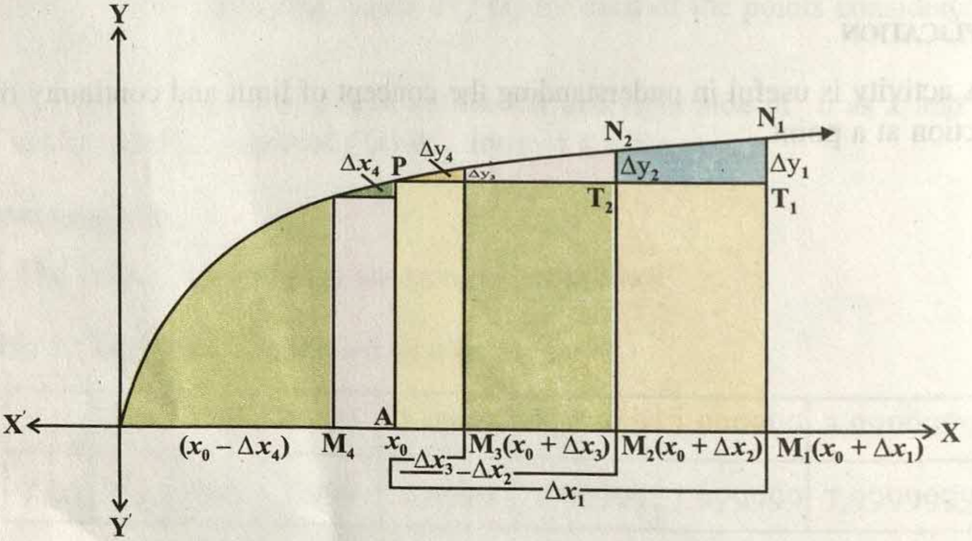


Fig. 10

DEMONSTRATION

1. Take one more point M_1 $(x_0 + \Delta x_1, 0)$ to the right of A, where Δx_1 is an increment in x.
2. Draw the perpendicular from M_1 to meet the curve at N_1. Let the coordinates of N_1 be $(x_0 + \Delta x_1, y_0 + \Delta y_1)$
3. Draw a perpendicular from the point P (x_0, y_0) to meet N_1M_1 at T_1.
4. Now measure $AM_1 = \Delta x_1$ (say) and record it and also measure $N_1T_1 = \Delta y_1$ and record it.
5. Reduce the increment in x to Δx_2 (i.e., $\Delta x_2 < \Delta x_1$) to get another point M_2 $(x_0 + \Delta x_2, 0)$. Get the corresponding point N_2 on the curve
6. Let the perpendicular PT_1 intersects N_2M_2 at T_2.
7. Again measure $AM_2 = \Delta x_2$ and record it.

   Measure $N_2T_2 = \Delta y_2$ and record it.
8. Repeat the above steps for some more points so that Δx becomes smaller and smaller.

OBSERVATION

S.No.	Value of increment in x_0	Corresponding increment in y
1.	$\lvert\Delta x_1\rvert =$ ________	$\lvert\Delta y_1\rvert =$ ________
2.	$\lvert\Delta x_2\rvert =$ ________	$\lvert\Delta y_2\rvert =$ ________
3.	$\lvert\Delta x_3\rvert =$ ________	$\lvert\Delta y_3\rvert =$ ________
4.	$\lvert\Delta x_4\rvert =$ ________	$\lvert\Delta y_4\rvert =$ ________
5.	$\lvert\Delta x_5\rvert =$ ________	$\lvert\Delta y_5\rvert =$ ________

6.	$\lvert\Delta x_6\rvert=$	$\lvert\Delta y_6\rvert=$
7.	$\lvert\Delta x_7\rvert=$	$\lvert\Delta y_7\rvert=$
8.	$\lvert\Delta x_8\rvert=$	$\lvert\Delta y_8\rvert=$
9.	$\lvert\Delta x_9\rvert=$	$\lvert\Delta y_9\rvert=$
10.		

2. So, Δy becomes __________ when Δx becomes smaller.

3. Thus $\lim\limits_{\Delta x \to 0} \Delta y = 0$ for a continuous function.

APPLICATION

This activity is helpful in explaining the concept of derivative (left hand or right hand) at any point on the curve corresponding to a function.

Activity 11

OBJECTIVE

To verify Rolle's Theorem.

MATERIAL REQUIRED

A piece of plywood, wires of different lengths, white paper, sketch pen.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent *x*-axis and *y*-axis (see Fig. 11).
3. Take a piece of wire of 15 cm length and bend it in the shape of a curve and fix it on the plywood as shown in the figure.

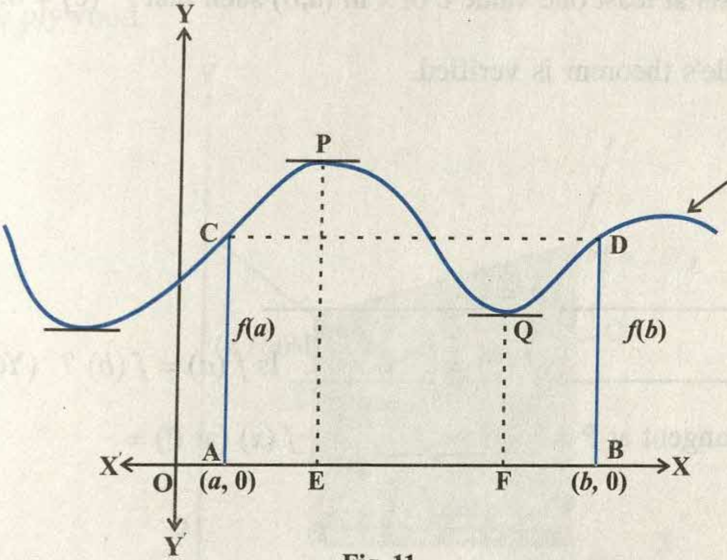


Fig. 11

4. Take two straight wires of the same length and fix them in such way that they are perpendicular to *x*-axis at the points A and B and meeting the curve at the points C and D (see Fig.11).

DEMONSTRATION

1. In the figure, let the curve represent the function $y = f(x)$. Let OA = a units and OB = b units.
2. The coordinates of the points A and B are $(a, 0)$ and $(b, 0)$, respectively.
3. There is no break in the curve in the interval $[a, b]$. So, the function f is continuous on $[a, b]$.
4. The curve is smooth between $x = a$ and $x = b$ which means that at each point, a tangent can be drawn which in turn gives that the function f is differentiable in the interval (a, b).
5. As the wires at A and B are of equal lengths, i.e., AC = BD, so $f(a) = f(b)$.
6. In view of steps (3), (4) and (5), conditions of Rolle's theorem are satisfied. From Fig.11, we observe that tangents at P as well as Q are parallel to x-axis, therefore, $f'(x)$ at P and also at Q are zero.

Thus, there exists at least one value c of x in (a,b) such that $f'(c) = 0$.

Hence, the Rolle's theorem is verified.

OBSERVATION

From Fig. 11.

$a =$ ______________, $b =$ ______________

$f(a) =$ ____________, $f(b) =$ __________ Is $f(a) = f(b)$? (Yes/No)

Slope of tangent at P = __________, so, $f(x)$ (at P) =

APPLICATION

This theorem may be used to find the roots of an equation.

Activity 12

OBJECTIVE

To verify Lagrange's Mean Value Theorem.

MATERIAL REQUIRED

A piece of plywood, wires, white paper, sketch pens, wires.

METHOD OF CONSTRUCTION

1. Take a piece of plywood and paste a white paper on it.
2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent *x*-axis and *y*-axis (see Fig. 12).
3. Take a piece of wire of about 10 cm length and bend it in the shape of a curve as shown in the figure. Fix this curved wire on the white paper pasted on the plywood.

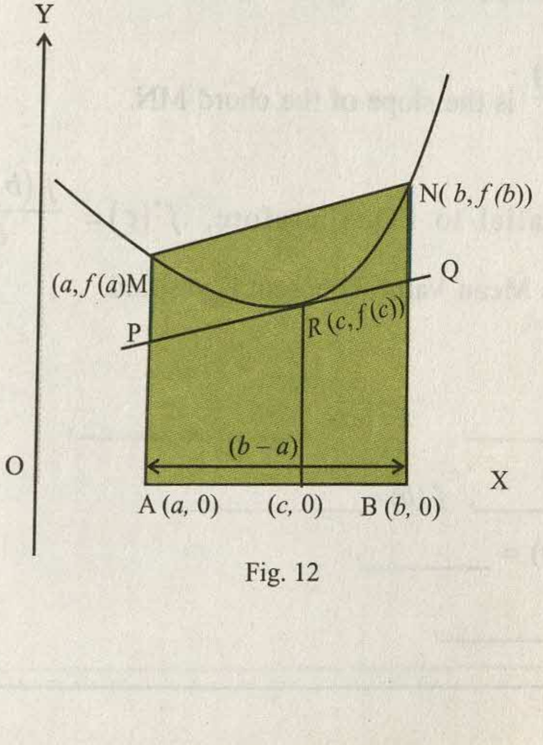


Fig. 12

4. Take two straight wires of lengths 10 cm and 13 cm and fix them at two different points of the curve parallel to y-axis and their feet touching the x-axis. Join the two points, where the two vertical wires meet the curve, using another wire.
5. Take one more wire of a suitable length and fix it in such a way that it is tangential to the curve and is parallel to the wire joining the two points on the curve (see Fig. 12).

DEMONSTRATION

1. Let the curve represent the function $y = f(x)$. In the figure, let OA = a units and OB = b units.
2. The coordinates of A and B are $(a, 0)$ and $(b, 0)$, respectively.
3. MN is a chord joining the points M $(a, f(a)$ and N $(b, f(b))$.
4. PQ represents a tangent to the curve at the point R $(c, f(c))$, in the interval (a, b).
5. $f'(c)$ is the slope of the tangent PQ at $x = c$.
6. $\dfrac{f(b)-f(a)}{b-a}$ is the slope of the chord MN.
7. MN is parallel to PQ, therefore, $f'(c) = \dfrac{f(b)-f(a)}{b-a}$. Thus, the Langrange's Mean Value Theorem is verified.

OBSERVATION

1. a = __________, b = ______________,

   $f(a)$ = _______, $f(b)$= __________.
2. $f(a) - f(b)$ = ________,

   $b - a$ = ________,

3. $\dfrac{f(b)-f(a)}{b-a}$ = ________ = Slope of MN.

4. Since PQ || MN $\Rightarrow$ Slope of PQ $= f'(c) = \dfrac{f(a)-f(a)}{b-a}$.

APPLICATION

Langrange's Mean Value Theorem has significant applications in calculus. For example this theorem is used to explain concavity of the graph.

Activity 13

OBJECTIVE

To understand the concepts of decreasing and increasing functions.

MATERIAL REQUIRED

Pieces of wire of different lengths, piece of plywood of suitable size, white paper, adhesive, geometry box, trigonometric tables.

METHOD OF CONSTRUCTION

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires of length say 20 cm each and fix them on the white paper to represent x-axis and y-axis.
3. Take two more pieces of wire each of suitable length and bend them in the shape of curves representing two functions and fix them on the paper as shown in the Fig. 13.

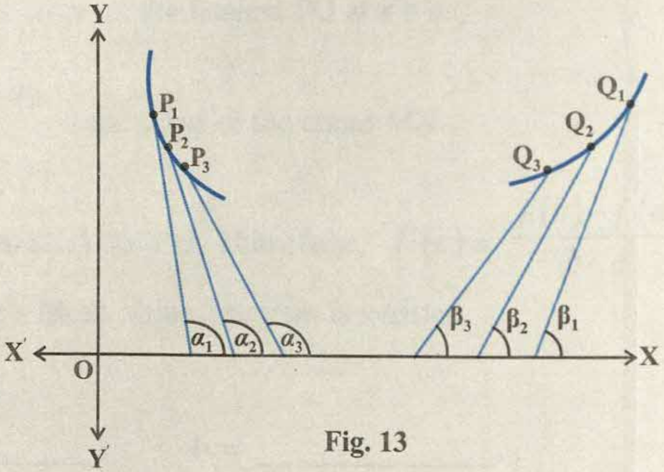


Fig. 13

4. Take two straight wires each of suitable length for the purpose of showing tangents to the curves at different points on them.

DEMONSTRATION

1. Take one straight wire and place it on the curve (on the left) such that it is

tangent to the curve at the point say P_1 and making an angle α_1 with the positive direction of *x*-axis.

2. α_1 is an obtuse angle, so tanα_1 is negative, i.e., the slope of the tangent at P_1 (derivative of the function at P_1) is negative.
3. Take another two points say P_2 and P_3 on the same curve, and make tangents, using the same wire, at P_2 and P_3 making angles α_2 and α_3, respectively with the positive direction of *x*-axis.
4. Here again α_2 and α_3 are obtuse angles and therefore slopes of the tangents tan α_2 and tan α_3 are both negative, i.e., derivatives of the function at P_2 and P_3 are negative.
5. The function given by the curve (on the left) is a decreasing function.
6. On the curve (on the right), take three point Q_1, Q_2, Q_3, and using the other straight wires, form tangents at each of these points making angles β_1, β_2, β_3, respectively with the positive direction of *x*-axis, as shown in the figure. β_1, β_2, β_3 are all acute angles.

   So, the derivatives of the function at these points are positive. Thus, the function given by this curve (on the right) is an increasing function.

OBSERVATION

1. α_1 = ______ , > 90° α_2 = ______ > ______, α_3 = ______ > ______, tan α_1 = ______, (negative) tan α_2 = ______, (______), tan α_3 = ______, (______). Thus the function is ______.
2. β_1 = ______ < 90°, β_2 = ______, < ______, β_3 = ______ , < ______

   tan β_1 = ______ , (positive), tan β_2 = ______, (______), tan β_3 = ______(______). Thus, the function is ______.

APPLICATION

This activity may be useful in explaining the concepts of decreasing and increasing functions.

Activity 14

OBJECTIVE

To understand the concepts of local maxima, local minima and point of inflection.

MATERIAL REQUIRED

A piece of plywood, wires, adhesive, white paper.

METHOD OF CONSTRUCTION

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires each of length 40 cm and fix them on the paper on plywood in the form of *x*-axis and *y*-axis.
3. Take another wire of suitable length and bend it in the shape of curve. Fix this curved wire on the white paper pasted on plywood, as shown in Fig. 14.

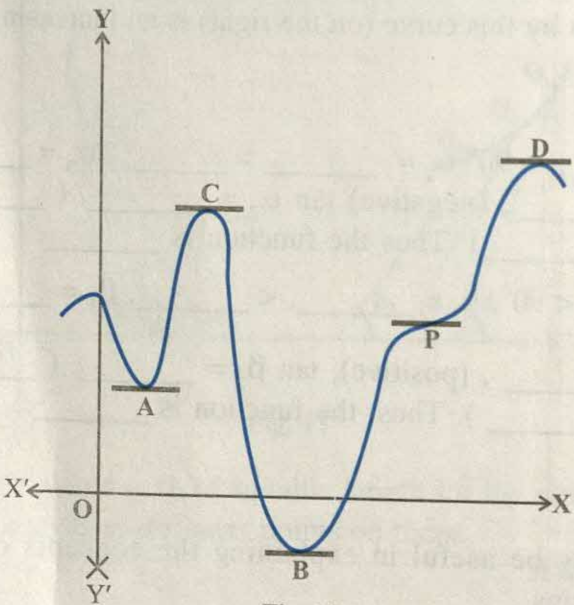


Fig. 14

4. Take five more wires each of length say 2 cm and fix them at the points A, C, B, P and D as shown in figure.

DEMONSTRATION

1. In the figure, wires at the points A, B, C and D represent tangents to the curve and are parallel to the axis. The slopes of tangents at these points are zero, i.e., the value of the first derivative at these points is zero. The tangent at P intersects the curve.
2. At the points A and B, sign of the first derivative changes from negative to positive. So, they are the points of local minima.
3. At the point C and D, sign of the first derivative changes from positive to negative. So, they are the points of local maxima.
4. At the point P, sign of first derivative does not change. So, it is a point of inflection.

OBSERVATION

1. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is ______.
2. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is______.
3. Sign of the first derivative at a point on the curve to immediate left of B is ______.
4. Sign of the first derivative at a point on the curve to immediate right of B is ______.
5. Sign of the first derivative at a point on the curve to immediate left of C is ______.
6. Sign of the first derivative at a point on the curve to immediate right of C is ______.
7. Sign of the first derivative at a point on the curve to immediate left of D is ______.

8. Sign of the first derivative at a point on the curve to immediate right of D is _______.

9. Sign of the first derivative at a point immediate left of P is _______ and immediate right of P is_______.

10. A and B are points of local _______.

11. C and D are points of local _______.

12. P is a point of _______.

APPLICATION

1. This activity may help in explaining the concepts of points of local maxima, local minima and inflection.
2. The concepts of maxima/minima are useful in problems of daily life such as making of packages of maximum capacity at minimum cost.

Activity 15

OBJECTIVE

To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

MATERIAL REQUIRED

Drawing board, white chart paper, adhesive, geometry box, pencil and eraser, sketch pens, ruler, calculator.

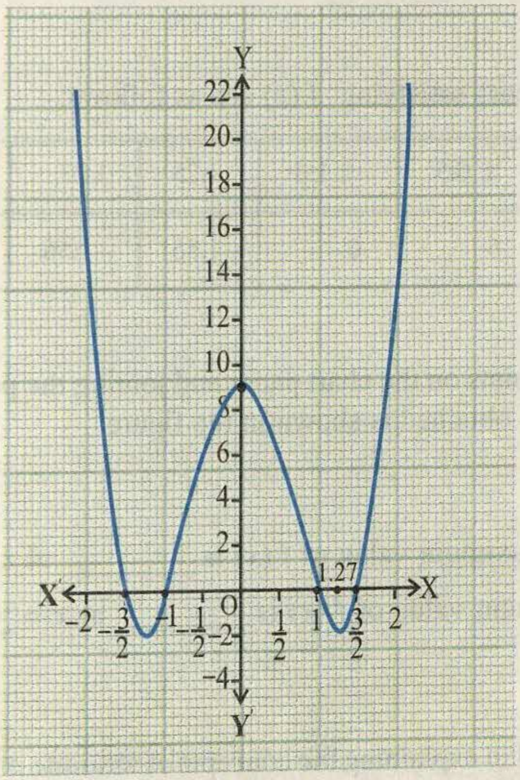


Fig 15

METHOD OF CONSTRUCTION

1. Fix a white chart paper of convenient size on a drawing board using adhesive.
2. Draw two perpendicular lines on the squared paper as the two rectangular axes.
3. Graduate the two axes as shown in Fig.15.
4. Let the given function be $f(x) = (4x^2 - 9)(x^2 - 1)$ in the interval $[-2, 2]$.
5. Taking different values of x in $[-2, 2]$, find the values of $f(x)$ and plot the ordered pairs $(x, f(x))$.
6. Obtain the graph of the function by joining the plotted points by a free hand curve as shown in the figure.

DEMONSTRATION

1. Some ordered pairs satisfying $f(x)$ are as follows:

x	0	± 0.5	± 1.0	1.25	1.27	± 1.5	± 2
$f(x)$	9	6	0	– 1.55	–1:56	0	21

2. Plotting these points on the chart paper and joining the points by a free hand curve, the curve obtained is shown in the figure.

OBSERVATION

1. The absolute maximum value of $f(x)$ is _______ at x = _______.
2. Absolute minimum value of $f(x)$ is _______ at x = _______.

APPLICATION

The activity is useful in explaining the concepts of absolute maximum / minimum value of a function graphically.

NOTE

Consider $f(x) = (4x^2 - 9)(x^2 - 1)$

$f(x) = 0$ gives the values of x as $\pm\frac{3}{2}$ and ± 1. Both these values of x lie in the given closed interval $[-2, 2]$.

$f'(x) = (4x^2 - 9)\,2x + 8x(x^2 - 1) = 16x^3 - 26x = 2x(8x^2 - 13)$

$f'(x) = 0$ gives $x = 0, x = \pm\sqrt{\frac{13}{8}} = \pm 1.27$. These two values of x lie in $[-2, 2]$.

The function has local maxima/minima at $x = 0$ and $x = \pm$ 1.27, respectively.

Activity 16

OBJECTIVE

To construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner.

MATERIAL REQUIRED

Chart papers, scissors, cellotape, calculator.

METHOD OF CONSTRUCTION

1. Take a rectangular chart paper of size 20 cm × 10 cm and name it as ABCD.
2. Cut four equal squares each of side x cm from each corner A, B, C and D.
3. Repeat the process by taking the same size of chart papers and different values of x.
4. Make an open box by folding its flaps using cellotape/adhesive.

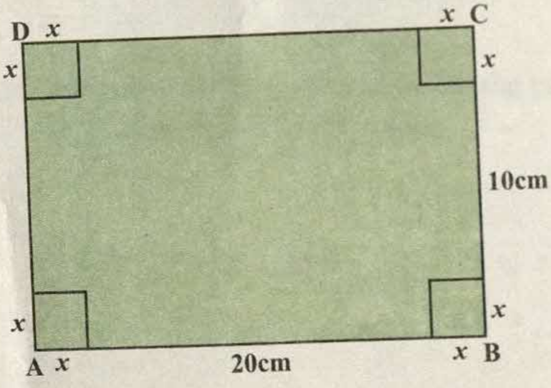


Fig. 16

DEMONSTRATION

1. When $x = 1$, Volume of the box = 144 cm^3
2. When $x = 1.5$, Volume of the box = 178.5 cm^3

3. When $x = 1.8$, Volume of the box = 188.9 cm^3.
4. When $x = 2$, Volume of the box = 192 cm^3.
5. When $x = 2.1$, Volume of the box = 192.4 cm^3.
6. When $x = 2.2$, Volume of the box = 192.2 cm^3.
7. When $x = 2.5$, Volume of the box = 187.5 cm^3.
8. When $x = 3$, Volume of the box = 168 cm^3.

Clearly, volume of the box is maximum when $x = 2.1$.

OBSERVATION

1. V_1 = Volume of the open box (when $x = 1.6$) = ...
2. V_2 = Volume of the open box (when $x = 1.9$) = ...
3. V = Volume of the open box (when $x = 2.1$) = ...
4. V_3 = Volume of the open box (when $x = 2.2$) = ...
5. V_4 = Volume of the open box (when $x = 2.4$) = ...
6. V_5 = Volume of the open box (when $x = 3.2$) = ...
7. Volume V_1 is ____________ than volume V.
8. Volume V_2 is ____________ than volume V.
9. Volume V_3 is ____________ than volume V.
10. Volume V_4 is ____________ than volume V.
11. Volume V_5 is ____________ than volume V.

So, Volume of the open box is maximum when x = ________.

APPLICATION

This activity is useful in explaining the concepts of maxima/minima of functions. It is also useful in making packages of maximum volume with minimum cost.

Let V denote the volume of the box.

Now $V = (20 - 2x)(10 - 2x)x$

or $V = 200x - 60x^2 + 4x^3$

$\frac{dV}{dx} = 200 - 120x + 12x^2$. For maxima or minima, we have,

$\frac{dV}{dx} = 0$, i.e., $3x^2 - 30x + 50 = 0$

i.e., $x = \frac{30 \pm \sqrt{900 - 600}}{6} = 7.9$ or 2.1

Reject $x = 7.9$.

$\frac{d^2V}{dx^2} = -120 + 24x$

When $x = 2.1$, $\frac{d^2V}{dx^2}$ is negative.

Hence, V should be maximum at $x = 2.1$.

Activity 17

OBJECTIVE

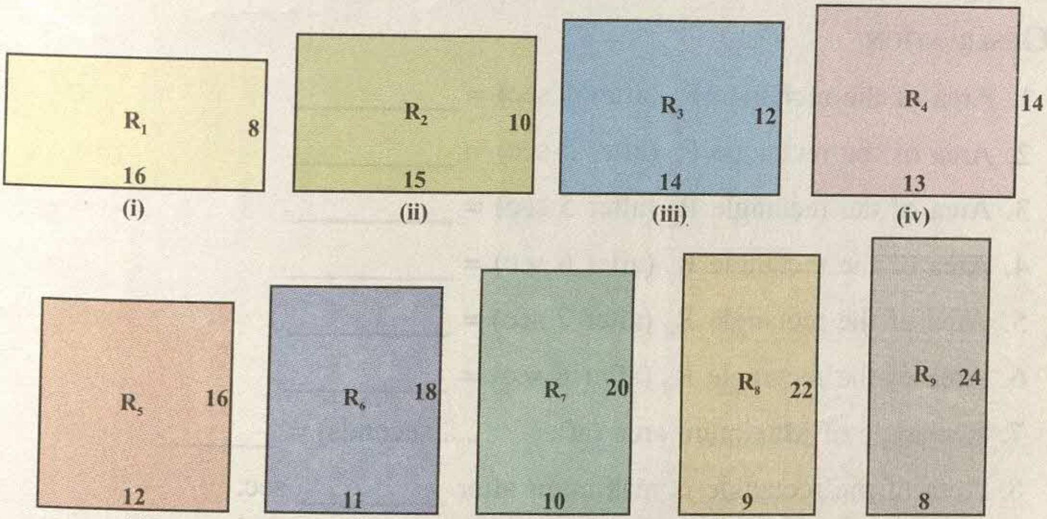
To find the time when the area of a rectangle of given dimensions become maximum, if the length is decreasing and the breadth is increasing at given rates.

MATERIAL REQUIRED

Chart paper, paper cutter, scale, pencil, eraser, cardboard.

METHOD OF CONSTRUCTION

1. Take a rectangle R_1 of dimensions 16 cm × 8 cm.
2. Let the length of the rectangle is decreasing at the rate of 1cm/second and the breadth is increasing at the rate of 2 cm/second.
3. Cut other rectangle R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, etc. of dimensions 15 cm × 10 cm, 14 cm × 12 cm, 13 cm × 14 cm, 12 cm × 16 cm, 11 cm × 18 cm, 10 cm × 20 cm, 9 cm × 22 cm, 8 cm × 24 cm (see Fig.17).
4. Paste these rectangles on card board.



(i) (ii) (iii) (iv)

(v) (vi) (vii) (viii) (ix)

Fig. 17

DEMONSTRATION

1. Length of the rectangle is decreasing at the rate of 1cm/s and the breadth is increasing at the rate of 2cm/s.
2. (i) Area of the given rectangle $R_1 = 16 \times 8 = 128$ cm^2.
   (ii) Area of rectangle $R_2 = 15 \times 10 = 150$ cm^2 (after 1 sec).
   (iii) Area of rectangle $R_3 = 168$ cm^2 (after 2 sec).
   (iv) Area of rectangle $R_4 = 182$ cm^2 (after 3 sec).
   (v) Area of rectangle $R_5 = 192$ cm^2 (after 4 sec).
   (vi) Area of rectangle $R_6 = 198$ cm^2 (after 5 sec).
   (vii) Area of rectangle $R_7 = 200$ cm^2 (after 6 sec).
   (viii) Area of rectangle $R_8 = 198$ cm^2 (after 7 sec) and so on.

Thus the area of the rectangle is maximum after 6 sec.

OBSERVATION

1. Area of the rectangle R_2 (after 1 sec) = ________.
2. Area of the rectangle R_4 (after 3 sec) = ________.
3. Area of the rectangle R_6 (after 5 sec) = ________.
4. Area of the rectangle R_7 (after 6 sec) = ________.
5. Area of the rectangle R_8 (after 7 sec) = ________.
6. Area of the rectangle R_9 (after 8 sec) = ________.
7. Rectangle of Maximum area (after ..... seconds) = ________.
8. Area of the rectangle is maximum after ________ sec.
9. Maximum area of the rectangle is ________.

APPLICATION

This activity can be used in explaining the concept of rate of change and optimisation of a function.

The function has local maxima/minima at $x = 0$ and $x = \pm 1.27$, respectively.

NOTE

Let the length and breadth of rectangle be a and b.

The length of rectangle after t seconds $= a - t$.

The breadth of rectangle after t seconds $= b + 2t$.

Area of the rectangle (after t sec) $= A(t) = (a - t)(b + 2t) = ab - bt + 2at - 2t^2$

$A'(t) = -b + 2a - 4t$

For maxima or minima, $A'(t) = 0$.

$$A'(t)=0 \Rightarrow t=\frac{2a-b}{4}$$

$$A''(t)=-4$$

$A''\left(\frac{2a-b}{4}\right)=-4$, which is negative

Thus, $A(t)$ is maximum at $t=\frac{2a-b}{4}$ seconds.

Here, a = 16 cm, b = 8 cm.

Thus, $t=\frac{32-8}{4}=\frac{24}{4}=6$ seconds

Hence, after 6 second, the area will become maximum.

Activity 18

OBJECTIVE

To verify that amongst all the rectangles of the same perimeter, the square has the maximum area.

MATERIAL REQUIRED

Chart paper, paper cutter, scale, pencil, eraser cardboard, glue.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Make rectangles each of perimeter say 48 cm on a chart paper. Rectangles of different dimensions are as follows:

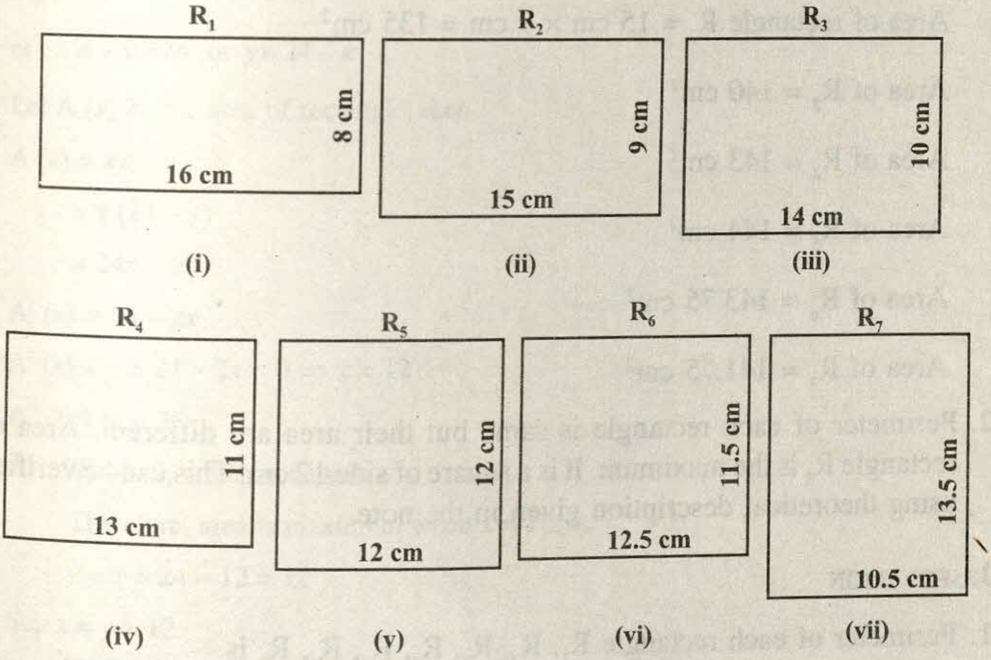


Fig. 18

R_1 : 16 cm × 8 cm, R_2 : 15 cm × 9 cm

R_3 : 14 cm × 10 cm, R_4 : 13 cm × 11 cm

R_5 : 12 cm × 12 cm, R_6 : 12.5 cm × 11.5 cm

R_7 : 10.5 cm × 13.5 cm

3. Cut out these rectangles and paste them on the white paper on the cardboard (see Fig. 18 (i) to (vii)).
4. Repeat step 2 for more rectangles of different dimensions each having perimeter 48 cm.
5. Paste these rectangles on cardboard.

DEMONSTRATION

1. Area of rectangle of $R_1 = 16 \text{ cm} \times 8 \text{ cm} = 128 \text{ cm}^2$

   Area of rectangle $R_2 = 15 \text{ cm} \times 9 \text{ cm} = 135 \text{ cm}^2$

   Area of $R_3 = 140 \text{ cm}^2$

   Area of $R_4 = 143 \text{ cm}^2$

   Area of $R_5 = 144 \text{ cm}^2$

   Area of $R_6 = 143.75 \text{ cm}^2$

   Area of $R_7 = 141.75 \text{ cm}^2$

2. Perimeter of each rectangle is same but their area are different. Area of rectangle R_5 is the maximum. It is a square of side 12 cm. This can be verified using theoretical description given in the note.

OBSERVATION

1. Perimeter of each rectangle R_1, R_2, R_3, R_4, R_4, R_6, R_7 is __________.
2. Area of the rectangle R_3 ________ than the area of rectangle R_5.

3. Area of the rectangle R_6 _______ than the area of rectangle R_5.
4. The rectangle R_5 has the diamensions ______ × ______ and hence it is a ________.
5. Of all the rectangles with same perimeter, the ________ has the maximum area.

APPLICATION

This activity is useful in explaining the idea of Maximum of a function. The result is also useful in preparing economical packages.

NOTE

Let the length and breadth of rectangle be x and y.

The perimeter of the rectangle P = 48 cm.

$2(x + y) = 48$

or $x + y = 24$ or $y = 24 - x$

Let A (x) be the area of rectangle, then

$$A(x) = xy$$
$$= x(24 - x)$$
$$= 24x - x^2$$
$$A'(x) = 24 - 2x$$
$$A'(x) = \Rightarrow 24 - 2x = 0 \Rightarrow x = 12$$
$$A''(x) = -2$$

$A''(12) = -2$, which is negative

Therefore, area is maximum when $x = 12$

$y = x = 24 - 12 = 12$

So, $x = y = 12$

Hence, amongst all rectangles, the square has the maximum area.

Activity 19

OBJECTIVE

To evaluate the definite integral $\int_a^b \sqrt{(1-x^2)}\,dx$ as the limit of a sum and verify it by actual integration.

MATERIAL REQUIRED

Cardboard, white paper, scale, pencil, graph paper

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw two perpendicular lines to represent coordinate axes XOX′ and YOY′.
3. Draw a quadrant of a circle with O as centre and radius 1 unit (10 cm) as shown in Fig.19.

The curve in the 1st quadrant represents the graph of the function $\sqrt{1-x^2}$ in the interval [0, 1].

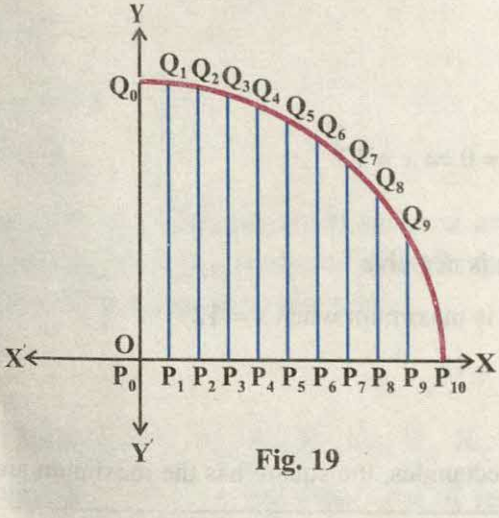


Fig. 19

DEMONSTRATION

1. Let origin O be denoted by P_0 and the points where the curve meets the x-axis and y-axis be denoted by P_{10} and Q, respectively.
2. Divide P_0P_{10} into 10 equal parts with points of division as, P_1, P_2, P_3, ..., P_9.
3. From each of the points, P_i, $i = 1, 2, ..., 9$ draw perpendiculars on the x-axis to meet the curve at the points, Q_1, Q_2, Q_3 ,..., Q_9. Measure the lengths of P_0Q_0, P_1Q_1, ..., P_9Q_9 and call them as $y_0, y_1, ..., y_9$ whereas width of each part, P_0P_1, P_1P_2, ..., is 0.1 units.
4. $y_0 = P_0Q_0 = 1$ units

   $y_1 = P_1Q_1 = 0.99$ units

   $y_2 = P_2Q_2 = 0.97$ units

   $y_3 = P_3Q_3 = 0.95$ units

   $y_4 = P_4Q_4 = 0.92$ units

   $y_5 = P_5Q_5 = 0.87$ units

   $y_6 = P_6Q_6 = 0.8$ units

   $y_7 = P_7Q_7 = 0.71$ units

   $y_8 = P_8Q_8 = 0.6$ units

   $y_9 = P_9Q_9 = 0.43$ units

   $y_{10} = P_{10}Q_{10} =$ which is very small near to 0.
5. Area of the quadrant of the circle (area bounded by the curve and the two axis) = sum of the areas of trapeziums.

$$= \frac{1}{2} \times 0.1 \begin{bmatrix} (1+0.99)+(0.99+0.97)+(0.97+0.95)+(0.95+0.92) \\ +(0.92+0.87)+(0.87+0.8)+(0.8+0.71)+(0.71+0.6) \\ +(0.6+0.43)+(0.43) \end{bmatrix}$$

$= 0.1\ [0.5 + 0.99 + 0.97 + 0.95 + 0.92 + 0.87 + 0.80 + 0.71 + 0.60 + 0.43]$

$= 0.1 \times 7.74 = 0.774$ sq. units.(approx.)

6. Definite integral = $\int_0^1 \sqrt{1-x^2}\, dx$

$$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1} x\right]_0^1 = \frac{1}{2} \times \frac{\pi}{2} = \frac{3.14}{4} = 0.785 \text{ sq.units}$$

Thus, the area of the quadrant as a limit of a sum is nearly the same as area obtained by actual integration.

OBSERVATION

1. Function representing the arc of the quadrant of the circle is $y =$ ______.
2. Area of the quadrant of a circle with radius 1 unit = $\int_0^1 \sqrt{1-x^2}\, dx =$ ________. sq. units
3. Area of the quadrant as a limit of a sum = ________ sq. units.
4. The two areas are nearly _________.

APPLICATION

This activity can be used to demonstrate the concept of area bounded by a curve. This activity can also be applied to find the approximate value of π.

NOTE

Demonstrate the same activity by drawing the circle $x^2 + y^2 = 9$ and find the area between $x = 1$ and $x = 2$.

Activity 20

OBJECTIVE

To verify geometrically that

$\vec{c}\times\left(\vec{a}+\vec{b}\right)=\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$

MATERIAL REQUIRED

Geometry box, cardboard, white paper, cutter, sketch pen, cellotape.

METHOD OF CONSTRUCTION

1. Fix a white paper on the cardboard.
2. Draw a line segment OA (= 6 cm, say) and let it represent $\vec{c}$.
3. Draw another line segment OB (= 4 cm, say) at an angle (say 60°) with OA.

   Let $\overrightarrow{OB}=\vec{a}$

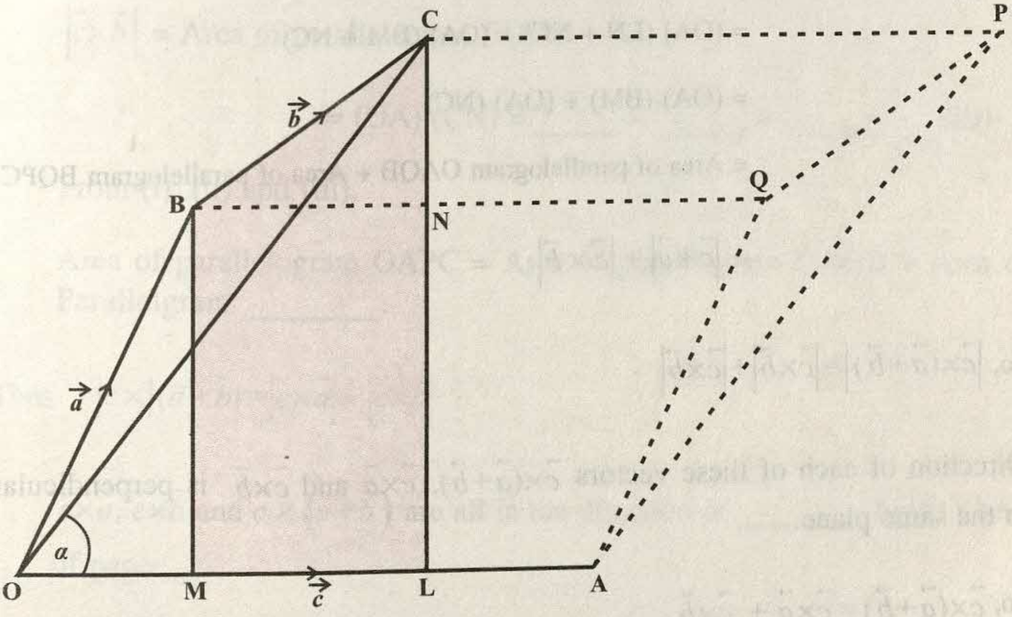


Fig. 20

4. Draw BC (= 3 cm, say) making an angle (say 30°) with $\overrightarrow{\mathrm{OA}}$. Let $\overrightarrow{\mathrm{BC}} = \vec{b}$

5. Draw perpendiculars BM, CL and BN.

6. Complete parallelograms OAPC, OAQB and BQPC.

DEMONSTRATION

1. $\overrightarrow{\mathrm{OC}} = \overrightarrow{\mathrm{OB}} + \overrightarrow{\mathrm{BC}} = \vec{a} + \vec{b}$, and let $\angle \mathrm{COA} = \alpha$.

2. $\left|\vec{c} \times \left(\vec{a} + \vec{b}\right)\right| = \left|\vec{c}\right| \left|\vec{a} + \vec{b}\right| \sin \alpha$ = area of parallelogram OAPC.

3. $\left|\vec{c} \times \vec{a}\right|$ = area of parallelogram OAQB.

4. $\left|\vec{c} \times \vec{b}\right|$ = area of parallelogram BQPC.

5. Area of parallelogram OAPC = (OA) (CL)

= (OA) (LN + NC) = (OA) (BM + NC)

= (OA) (BM) + (OA) (NC)

= Area of parallelogram OAQB + Area of parallelogram BQPC

$= \left|\vec{c} + \vec{a}\right| + \left|\vec{c} \times \vec{b}\right|$

So, $\left|\vec{c} \times (\vec{a} + \vec{b})\right| = \left|\vec{c} \times \vec{b}\right| + \left|\vec{c} \times \vec{b}\right|$

Direction of each of these vectors $\vec{c} \times (\vec{a} + \vec{b})$, $\vec{c} \times \vec{a}$ and $\vec{c} \times \vec{b}$ is perpendicular to the same plane.

So, $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

OBSERVATION

$|\vec{c}| = |\overrightarrow{\mathrm{OA}}| = \mathrm{OA} =$ ______

$|\vec{a}+\vec{b}| = |\overrightarrow{\mathrm{OC}}| = \mathrm{OC} =$ ______

CL = ______

$|\vec{c}\times(\vec{a}+\vec{b})|$ = Area of parallelogram OAPC

= (OA) (CL) = ____________ sq. units (i)

$|\vec{c}\times\vec{a}\,|$ = Area of parallelogram OAQB

= (OA) (BM) = _____ × _____ = ______ (ii)

$|\vec{c}\times\vec{b}\,|$ = Area of parallelogram BQPC

= (OA) (CN) = _____ × _____ = ______ (iii)

From (i), (ii) and (iii),

Area of parallelogram OAPC = Area of parallelgram OAQB + Area of Parallelgram ________.

Thus $|\bar{c}\times|(\bar{a}+\bar{b}| = |\vec{c}\times\vec{a}| + |\vec{c}\times\vec{b}|$

$\vec{c}\times\vec{a}$, $\vec{c}\times\vec{b}$ and $\vec{c}\times\left(\vec{a}+\vec{b}\right)$ are all in the direction of ______ to the plane of paper.

Therefore $\vec{c}\times\left(\vec{a}+\vec{b}\right)=\vec{c}\times\vec{a}+$ ________.

APPLICATION

Through the activity, distributive property of vector multiplication over addition can be explained.

NOTE

This activity can also be performed by taking rectangles instead of parallelograms.

Activity 21

OBJECTIVE

To verify that angle in a semi-circle is a right angle, using vector method.

MATERIAL REQUIRED

Cardboard, white paper, adhesive, pens, geometry box, eraser, wires, paper arrow heads.

METHOD OF CONSTRUCTION

1. Take a thick cardboard of size 30 cm × 30 cm.
2. On the cardboard, paste a white paper of the same size using an adhesive.
3. On this paper draw a circle, with centre O and radius 10 cm.

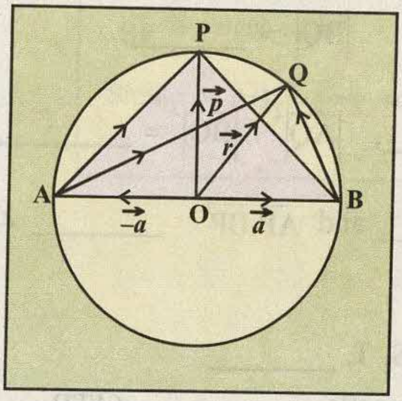


Fig. 21

4. Fix nails at the points O, A, B, P and Q. Join OP, OA, OB, AP, AQ, BQ, OQ and BP using wires.
5. Put arrows on OA, OB, OP, AP, BP, OQ, AQ and BQ to show them as vectors, using paper arrow heads, as shown in the figure.

DEMONSTRATION

1. Using a protractor, measure the angle between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$, i.e., $\angle APB = 90°$.

2. Similarly, the angle between the vectors $\overrightarrow{AQ}$ and $\overrightarrow{BQ}$, i.e., ∠ AQB = 90°.
3. Repeat the above process by taking some more points R, S, T, ... on the semi-circles, forming vectors AR, BR; AS, BS; AT, BT; ..., etc., i.e., angle formed between two vectors in a semi-circle is a right angle.

OBSERVATION

By actual measurement.

$|\overrightarrow{OP}| = |\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OQ}| = r = a = p =$ ________ ,

$|\overrightarrow{AP}| =$ ________ , $|\overrightarrow{BP}| =$ ________ , $|\overrightarrow{AB}| =$ ________

$|\overrightarrow{AQ}| =$ ________ , $|\overrightarrow{BQ}| =$ ________

$|\overrightarrow{AP}|^2 + |\overrightarrow{BP}|^2 =$ ________ , $|\overrightarrow{AQ}|^2 + |\overrightarrow{BQ}|^2 =$ ________

So, ∠APB = ________ and $\overrightarrow{AP}.\overrightarrow{BP}$ ________ ∠AQB = ________ and

$\overrightarrow{AQ}.\overrightarrow{BP}$ = ________

Similarly, for points R, S, T, ________

∠ARB = ________ , ∠ASB = ________ , ∠ATB = ________ , ________

i.e., angle in a semi-circle is a right angle.

APPLICATION

This activity can be used to explain the concepts of

(i) opposite vectors

(ii) vectors of equal magnitude

(iii) perpendicular vectors

(iv) Dot product of two vectors.

Let OA = OB = a = OP = p

$\overrightarrow{OA} = -\vec{a}$, $\overrightarrow{OB} = \vec{a}$, $\overrightarrow{OP} = \vec{p}$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP} = \vec{a} + \vec{p}$., $\overrightarrow{BP} = \vec{p} - \vec{a}$.

$\overrightarrow{AP}.\ \overrightarrow{BP} = (\vec{p} + \vec{a}).(\vec{p} - \vec{a}) = |\vec{p}|^2 - |\vec{a}|^2 = 0$

$\left(\text{since} |\vec{p}|^2 = |\vec{a}|\right)$

So, the angle APB between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ is a right angle.

Similarly, $\overrightarrow{AQ}.\ \overrightarrow{BQ} = 0$, so, $\angle AQB = 90°$ and so on.

Activity 22

OBJECTIVE

To locate the points to given coordinates in space, measure the distance between two points in space and then to verify the distance using distance formula.

MATERIAL REQUIRED

Drawing board, geometry box, squared paper, nails of different lengths, paper arrows.

METHOD OF CONSTRUCTION

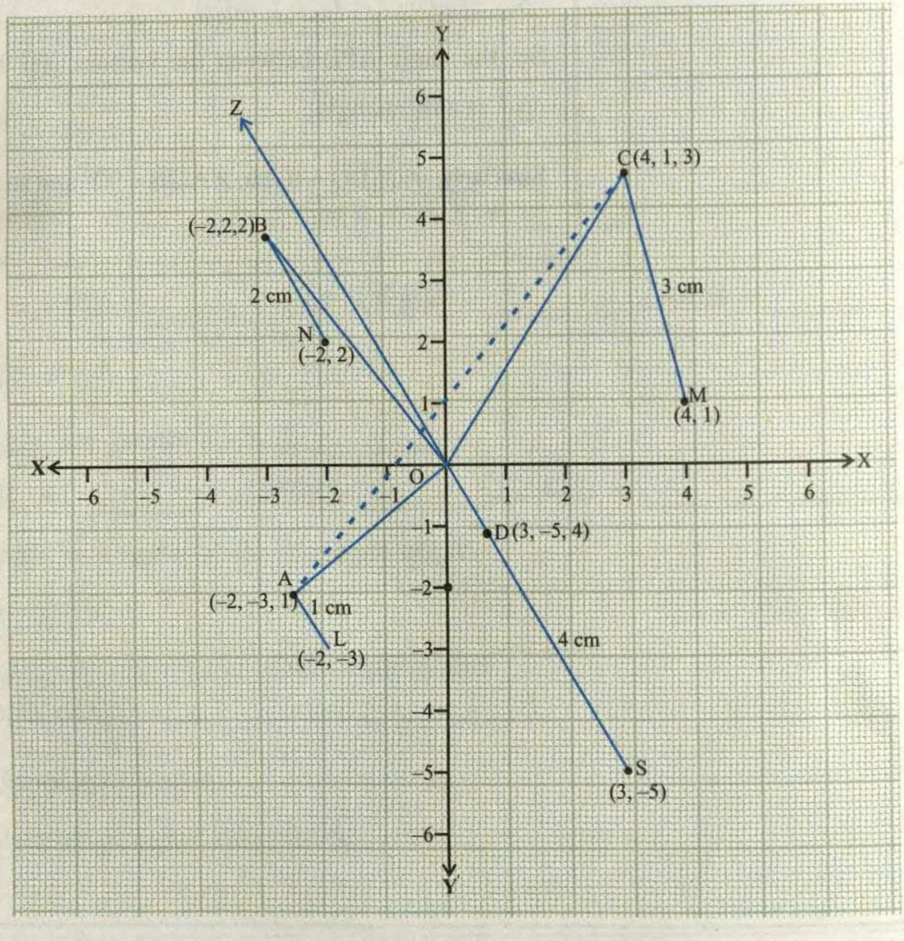


Fig 22

1. Take a drawing board and paste a squared paper on it.
2. Draw two lines X′OX and Y′OY to represent x-axis, y-axis respectively (see Fig. 22) and take 1 unit = 1 cm.
3. Fix a wire through O, in the vertical direction, representing the z-axis.
4. Fix nails of length 1 cm, 2 cm, 3 cm, 4 cm, etc. at different points on the squared paper (say at L (–2, –3), N (–2, 2), M (4, 1), S (3, –5)), etc.

Now the upper tips of these nails represent the points (say A, B, C, D) in the space.

DEMONSTRATION

1. Coordinates of the point A = (–2, –3, 1).
2. Coordinates of the point B = (–2, 2, 2).
3. Similarly find the coordinates of the point C and D.
4. By actual measurement (using a scale) distance AB = 5.1 cm.
5. By distance formula, $AB=\sqrt{(-2+2)^2+(-3-2)^2+(1-2)^2} = \sqrt{26} = 5.099$.

Thus, the distance AB, obtained by actual measurement is approximately same as the distance obtained by using the distance formula.

Same can be verified for other pairs of points A, C; B, C; A, D; C, D; B, D.

OBSERVATION

Coordinates of the point C = ________.

Coordinates of the point D = ________.

On actual measurement :

AC = ________, BC = ________.

AD = ________, CD = ________, BD = ________.

Using distance formula; AC = ________, BC = ________, AD = ________

CD = ________, BD = ________.

Thus, the distance between two points in space obtained on actual measurement and by using distance formula is approximately the same.

APPLICATION

1. This activity is useful in visualising the position of different points in space (coordinates of points).
2. The concept of position vectors can also be explained through this activity.

Activity 23

OBJECTIVE

To demonstrate the equation of a plane in normal form.

MATERIAL REQUIRED

Two pieces of plywood of siz 10 cm × 12 cm, a thin wooden rod with nuts and bolts fixed on both sides, 3 pieces of wires, pen/pencil.

METHOD OF CONSTRUCTION

1. Fix the wooden rod in between two wooden pieces with nuts and bolts so that the rod is perpendicular to the two wooden pieces. So, it represents the normal to the plane.
2. Take three wires and fix, them as shown in Fig. 23 so that $\overrightarrow{OP}$ represents the vector $\vec{a}$ and $\overrightarrow{OA}$ represents $\vec{r}$. Then the wire $\overrightarrow{PA}$ represents the vector $\vec{r}-\vec{a}$.

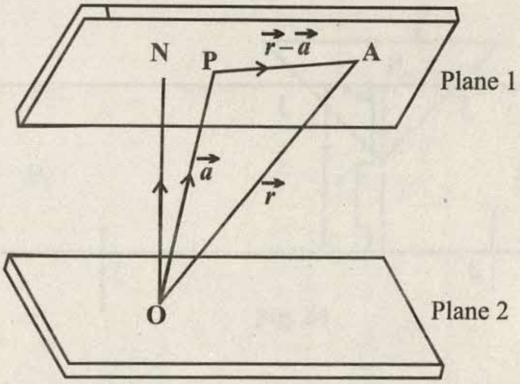


Fig. 23

DEMONSTRATION

1. The wire PA, i.e., the vector $(\vec{r}-\vec{a})$ lies on plane 1. On representing $\vec{n}$ as normal to plane 1, $\hat{n}$ is perpendicular to $(\vec{r}-\vec{a})$, normal to the plane.
2. Hence $\left(\vec{r}-\vec{a}\right)\cdot\vec{n}=0$ which gives the equation of plane in the normal form.

OBSERVATION

1. $\vec{a}$ is the position vector of ________, $\vec{r}$ is the position vector of ________ vector $\vec{n}$ is perpendicular to the vector ________.

2. $(\vec{r}-\vec{a}).\ \hat{n}=0$, is the equation of the plane ________, in ________ form.

APPLICATION

This activity can also be utilised to show the position vector of a point in space (i.e., $\vec{a}$ as position vector of O, $\vec{r}$ the position vector of A).

Activity 24

OBJECTIVE

To verify that the angle between two planes is the same as the angle between their normals.

MATERIAL REQUIRED

Plywood pieces, wires, hinges.

METHOD OF CONSTRUCTION

1. Take two pieces of plywood 10 cm × 20 cm and join them with the help of hinges.
2. Fix two vertical wires on each plane to show normals to the planes.
3. Cut slots in the two planes to fix a third plywood piece showing third plane.

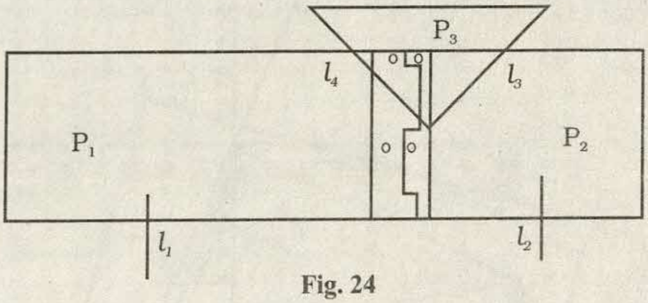


Fig. 24

DEMONSTRATION

1. P_1 represents the first plane.
2. P_2 represents the second plane.
3. Vertical wires l_1 and l_2 represents normals to the planes P_1, P_2, respectively.

4. l_3 and l_4 are the lines of intersections of the planes P_3, with P_1 and P_2, respectively.

5. Angle between lines l_3 and l_4 is the angle between the planes. It is same as the angle between their normals.

OBSERVATION

1. P_1 represents the ________.
2. P_2 represents the ________.
3. l_1 represents the ________.
4. l_2 represents the ________.
5. l_3 is the line of intersection ________.
6. l_4 is the line of intersection ________.
7. Angle between l_1 and l_2 is equal to ________.

APPLICATION

This model can also be used to find the angle between a line and a plane.

Activity 25

OBJECTIVE

To find the distance of given point (in space) from a plane (passing through three non-collinear points) by actual measurement and also analytically.

MATERIAL REQUIRED

One cardboard of size 20 cm × 30 cm and another of size 10 cm × 15 cm., a thick sheet of paper of size 20 cm × 30 cm, nails of varying lengths with caps on one end, geometry box, wires.

METHOD OF CONSTRUCTION

1. Draw two mutually perpendicular lines X′OX and Y′OY on a thick sheet of paper representing *x*-axis, and *y*-axis, respectively intersecting at O, and graduate them.

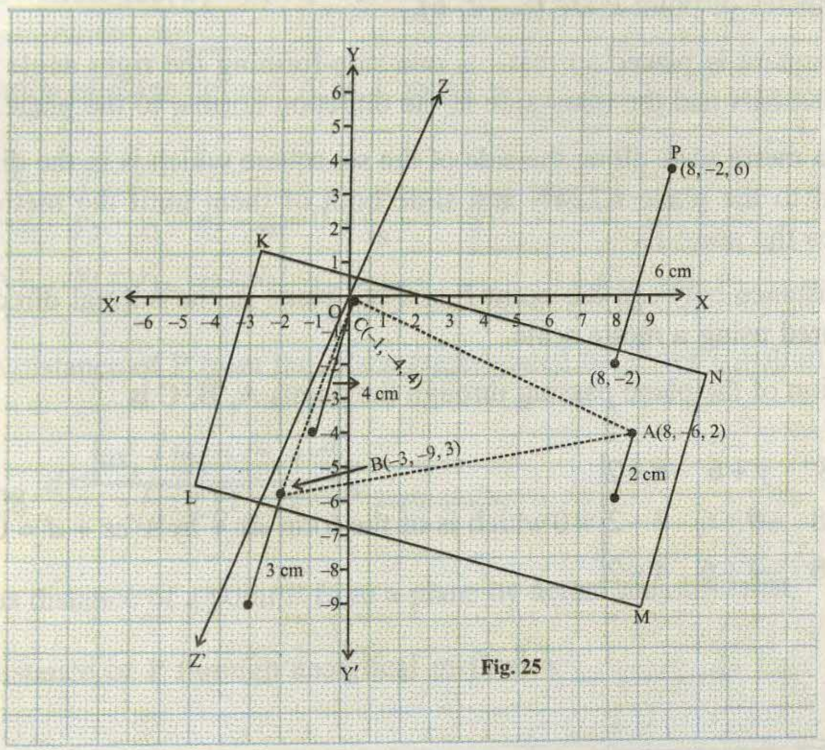


Fig. 25

2. Paste this sheet on the cardboard of size 20 cm × 30 cm. Through O, fix a wire vertically to represent z-axis (see Fig. 25).
3. Fix three nails of heights (say 2 cm, 3 cm and 4 cm) at three different points on this board (say at (8, –6), (–3, –9) and (–1, –4), respectevely) (Fig. 25).
4. The tips of these nails represent three points A, B and C in space.
5. Now rest a plane KLMN represented by another cardboard on the tips of these three nails so that the points A, B, C, lie on the plane.
6. Now fix a nail of length 6 cm at some point [say (8, –2)] on the cardboard. The tip of the nail will represent point P, from where the distance to the plane KLMN is to be found.

DEMONSTRATION

1. Coordinates of the points A, B and C are (8, –6, 2), (–3, –9, 3), (–1, – 4, 4), respectively.
2. Coordinates of point P are (8, –2, 6).
3. A set square is placed so that its one side-forming the right angle on the plane KLMN and the other side in the direction normal to the plane.
4. Place a metre scale along the side of the set square which is in the direction normal to the plane KLMN and slide both of them until the metre scale touches the point P.
5. The distance between the point P and the plane in the normal direction is measured using a metre scale.
6. Equation of the plane passing through the points A, B, C is

$$\begin{vmatrix} x-8 & y+6 & z-2 \\ -3-8 & -9+6 & 3-2 \\ -1-8 & -4+6 & 4-2 \end{vmatrix} = 0 \text{ which is of the form } ax + by + cz + d = 0.$$

7. This distance is also calculated by using the formula

$$d = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|.$$

8. The two distance so obtained are the same.

OBSERVATION

1. The coordinates of A (x_1, y_1, z_1) = ________.

B (x_2, y_2, z_2) = ________.

C (x_3, y_3, z_3) = ________.

Coordinates of point P = ________.

The distance(d) of P from the plane KLMN by actual measurement =________.

2. Equation of plane through A, B, C using

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$ is ________.

The distance of P from this plane (represented by above equation)

using $d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ = ________.

Thus distance of a point P from a plane by actual measurement

= distance of P through analytical method = ________.

APPLICATION

1. With this activity it can be explained that through
   (a) one point or through two points, infinite number of planes can pass,
   (b) three non-collinear points, a unique plane passes.
2. This activity can also be used in explainiing the concept of distance between two parallel planes.

Activity 26

OBJECTIVE

To measure the shortest distance between two skew lines and verify it analytically.

MATERIAL REQUIRED

A piece of plywood of size 30 cm × 20 cm, a squared paper, three wooden blocks of size 2cm × 2 cm × 2 cm each and one wooden block of size 2 cm × 2 cm × 4 cm, wires of different lengths, set squares, adhesive, pen/pencil, etc.

METHOD OF CONSTRUCTION

1. Paste a squared paper on a piece of plywood.
2. On the squared paper, draw two lines OA and OB to represent *x*-axis, and *y*-axis, respectively.
3. Name the three blocks of size 2 cm × 2 cm × 2 cm as I, II and III. Name the other wooden block of size 2 cm × 2 cm × 4 cm as IV.
4. Place blocks I, II, III such that their base centres are at the points (2, 2), (1, 6) and (7, 6), respectively, and block IV with its base centre at (6, 2). Other wooden block of size 2 cm × 2 cm × 4 cm as IV.
5. Place a wire joining the points P and Q, the centres of the bases of the blocks I and III and another wire joining the centres R and S of the tops of blocks II and IV as shown in Fig. 26.
6. These two wires represent two skew lines.
7. Take a wire and join it perpendicularly with the skew lines and measure the actual distance.

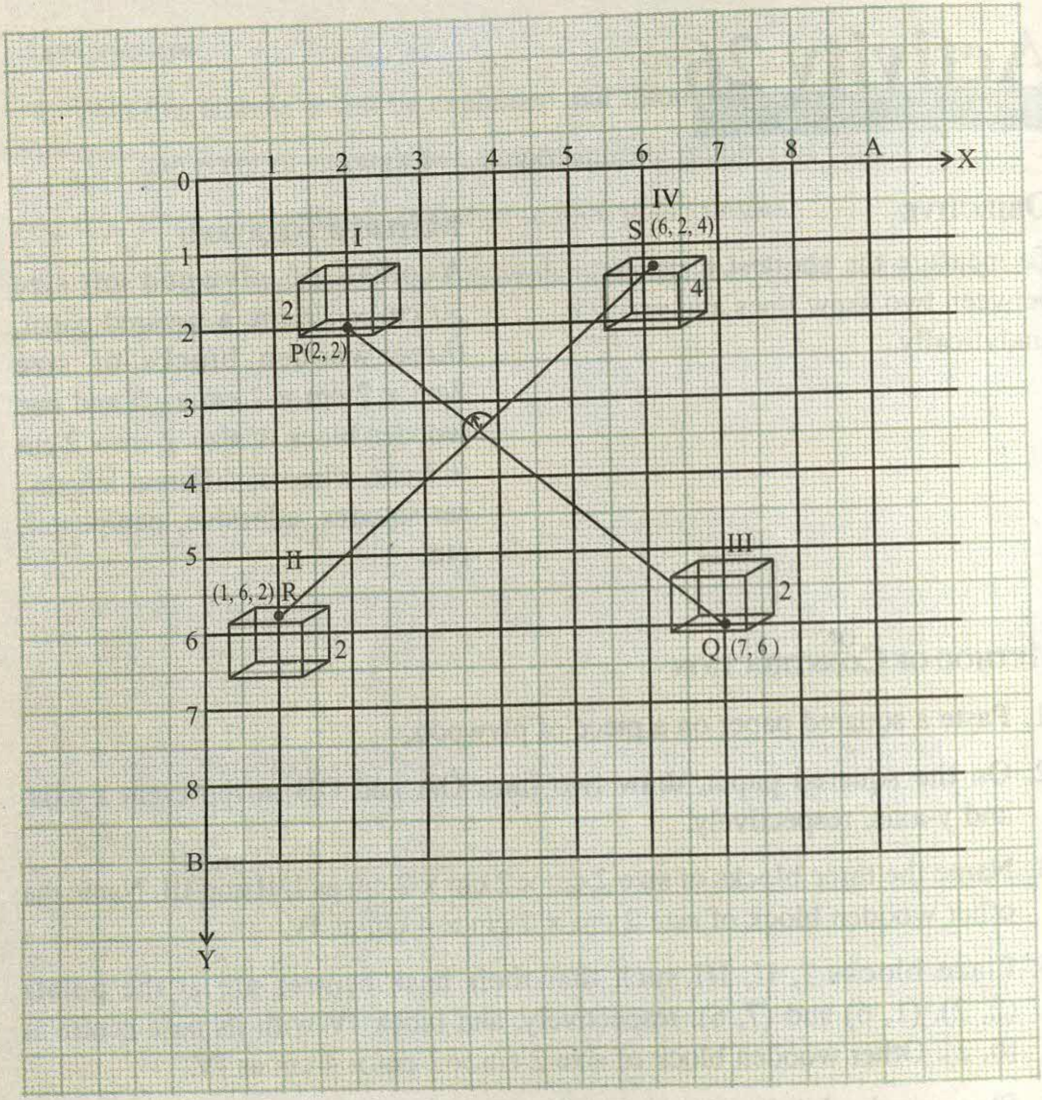


Fig. 26

DEMONSTRATION

1. A set-square is placed in such a way that its one perpendicular side is along the wire PQ.
2. Move the set-square along PQ till its other perpendicular side touches the other wire.

3. Measure the distance between the two lines in this position using set-square. This is the shortest distance between two skew lines.
4. Analytically, find the equation of line joining P (2, 2, 0) and Q (7, 6, 0) and other line joining R (1, 6, 2) and S (6, 2, 4) and find S.D. using $\frac{\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)\cdot\left(b_1\times b_2\right)}{\left|b_1\times\vec{b}_2\right|}$. The distance obtained in two cases will be the same.

OBSERVATION

1. Coordinates of point P are ________.
2. Coordinates of point Q are ________.
3. Coordinates of point R are ________.
4. Coordinates of point S are ________.
5. Equation of line PQ is ________.
6. Equation of line RS is ________.

Shortest distance between PQ and RS analytically = ________.

Shortest distance by actual measurement = ________.

The results so obtained are ________.

APPLICATION

This activity can be used to explain the concept of skew lines and of shortest distance between two lines in space.

Activity 27

OBJECTIVE

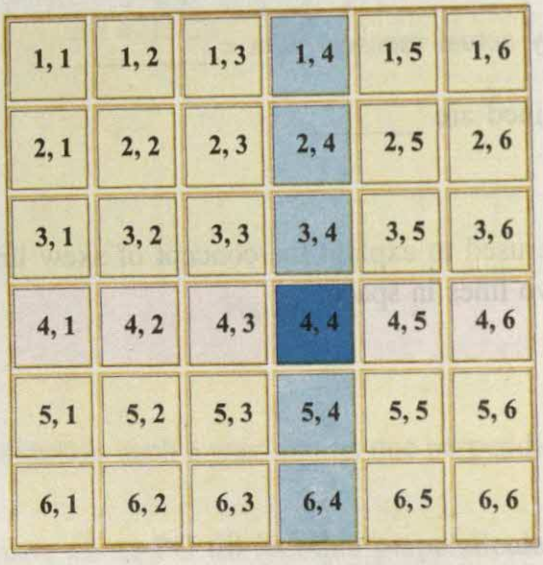
To explain the computation of conditional probability of a given event A, when event B has already occurred, through an example of throwing a pair of dice.

MATERIAL REQUIRED

A piece of plywood, white paper pen/pencil, scale, a pair of dice.

METHOD OF CONSTRUCTION

1. Paste a white paper on a piece of plywood of a convenient size.
2. Make a square and divide it into 36 unit squares of size 1cm each (see Fig. 27).
3. Write pair of numbers as shown in the figure.



1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Fig. 27

DEMONSTRATION

1. Fig. 27 gives all possible outcomes of the given experiment. Hence, it represents the sample space of the experiment.
2. Suppose we have to find the conditional probability of an event A if an event B has already occurred, where A is the event "a number 4 appears on both the dice" and B is the event "4 has appeared on at least one of the dice"i.e, we have to find P(A | B).
3. From Fig. 27 number of outcomes favourable to A = 1

   Number of outcomes favourable to B = 11

   Number of outcomes favourable to $A \cap B = 1$.

4. (i) $P(B) = \frac{11}{36}$,

   (ii) $P(A \cap B) = \frac{1}{36}$

   (iii) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{11}$.

NOTE

1. You may repeat this activity by taking more events such as the probability of getting a sum 10 when a doublet has already occurred.
2. Conditional probability P (A | B) can also be found by first taking the sample space of event B out of the sample space of the experiment, and then finding the probability A from it.

OBSERVATION

1. Outcome(s) favourable to A : ________, $n(A)$ = ________.
2. Outcomes favourable to B : ________, $n(B)$ = ________.
3. Outcomes favourable to $A \cap B$: ________, $n(A \cap B)$ = ________.
4. $P(A \cap B)$ = ________.
5. $P(A \mid B)$ = ________ = ________.

APPLICATION

This activity is helpful in understanding the concept of conditional probability, which is further used in Bayes' theorem.

Projects

Project work in mathematics may be performed individually by a student or jointly by a group of students. These projects may be in the form of construction such as curve sketching or drawing of graphs, etc. It may offer a discussion of a topic from history of mathematics involving the historical development of particular subject in mathematics/ topics on concepts. Students may be allowed to select the topics of their own choice for projects in mathematics. The teacher may act as a facilitator by creating interest in various topics. Once the topic has been selected, the student should read as much about the topic as is available and finally prepare the project.

Project 1

To minimise the cost of the food, meeting the dietary requirements of the staple food of the adolescent students of your school.

Task to be done

(i) Make a survey of atleast 100 students to find which staple food they consume on daily basis.

(ii) Select two food items constituting one cereal and one pulse.

(iii) Find from dietician the minimum requirement of protein and carbohydrate for an adolescent and also find the content of protein and carbohydrate in 1 kg. of selected cereal and pulse respectively.

(iv) Find the minimum cost of the selected cereal and pulse from market.

(v) Formulate the corresponding Linear Programming problem.

(vi) Solve the problem graphically.

(vii) Interpret the result.

Project 2

Estimation of the population of a particular region/country under the assumptions that there is no migration in or out of the existing population in a particular year.

Task to be done

1. Find the population of a selected region in a particular year.
2. Find the number of births and number of deaths in the existing population in a particular year t (say). Let

   $P(t)$: denote the population in a particular year t

   $B(t)$: denote the number of births in one year between t and $t + 1$.

   $D(t)$: denote the number of deaths in one year between t and $t + 1$.
3. Obtain the relation

   $$P(t+1) = P(t) + B(t) - D(t) \tag{1}$$
4. Assume that

   $b = \frac{B(t)}{P(t)}$ represents the birth rate for the time interval t to $t + 1$.

   $d = \frac{D(t)}{P(t)}$ represents death rate for the time interval t to $t + 1$.
5. From (1), we have

   $$\begin{aligned} P(t+1) &= P(t) + B(t) - D(t) \\ &= P(t)\left[1 + \frac{B(t)}{P(t)} - \frac{D(t)}{P(t)}\right] \\ &= P(t)(1 + b - d) \end{aligned} \tag{2}$$

6. Taking $t = 0$ in equation (2), we get

$$P(1) = P(0)(1 + b - d).$$

For $t = 1$, we get

$$P(2) = P(0)(1 + b - d)^2.$$

Continuing above equation, we get

$$P(t) = P(0)(1 + b - d)^t \qquad (3)$$

Here, it is assumed that birth rate and death rate remains the same for consecutive years. P(0) denote the initial population. Equation (3) gives the mathematical model for calculation the population in t year.

7. Using calculator find the population in different number of years.

8. Compare the population data obtained theoretically and draw the inferences.

Project 3

Finding the coordinates of different points identified in your classroom using the concepts of three dimensional geometry and also find the distances between the identified points.

Tasks to be done

1. Choose any corner of your classroom as the origin.
2. Take three perpendicular edges of walls as *x*–, *y*– and *z*-axes.
3. Find the coordinates of each corner of the room, corners of windows, doors and blackboard etc.
4. Find the coordinate of the tips of ceiling fan, bulbs and all other possible points in the space of the classroom.
5. Find the distances between different points by measurement as well as by using distance formula.
6. Find the coordinates of the diagonals of the room and length of the diagonals by distance formula.

Project 4

Formation of differential equation to explain the process of cooling of boiled water to a given room temperature.

Task to be done

1. Boil 1 litre of water in a pan/beaker.
2. Note the room temperature and the temperature of the boiled water.
3. Note the temperature at an interval of every half hour till the temperature of the water reaches the room temperature. Prepare a corresponding table as shown below:

Time (t) at an interval of $\frac{1}{2}$ hour	Temperature of water (T)	Room Temperature (P)	Difference T − P

4. Let T denote the temperature of the boiled water at time t. P denote the room temperature under the assumption it remains constant throughout the experiment.

$$\frac{dT}{dt} \alpha \, T - P.$$

or $\frac{dT}{dt} = -k\,(T - P)$, k is proportionality constant and minus sign signifier that temperature is decreasing.

or $\frac{dT}{T-P} = -k\,dt$. Integrating, we have

$$\log |T - P| = -kt + C \qquad (1)$$

5. Find the value of C and k by using two initial values of T and t from the observation table to get the particular solution of the differential equation (1).

List of Projects

1. Project on history of Mathematicians: It may include history of Indian mathematicians such as Aryabhata, Brahmgupta, Varahamihir, Sridhara, Bhaskaracharya, Ramanujan etc., and history of foreign mathematicians such as Cantor, Pythagoras, Thales, Euclid, Appollonius, Descartes, Fermat, Leibnitz, Euler, Fibonac, Gauss, Newton, etc.
2. On linear Programming problems related to day-to-day life like collecting data from families of their expenditures and requirements from the factories to maximum out put.
3. Collect data from dieticians, transporters, agents and formulate linear programming problems.
4. Make a chart of the formulae of applications of calculus.
5. Applications of conic sections, vectors, three dimensional geometry, calculus, etc., in Mathematics and Physics.
6. Mathematics and Chemistry: Study structure of organic compounds.
7. Mathematics and Biology: Study of science of heredity etc.
8. Mathematics and Music
9. Mathematics and Environment
10. Mathematics and Arts: Construction of shapes using curves
11. Mathematics and Information and Communication Technology: Writing of Mathematical programmes, flow charts, algorithm, circuit diagrams etc.
12. Collection of statistical data and analysing it for standard deviation and mean deviation.
13. Observe the various patterns and properties in Pascal's triangle and make a project.

14. Prepare a project based on the Fibonacci sequence, their properties and similar pattern found in nature.

15. Form a differential equation for the growth of bacteria in different environments.

16. Study the nature of mathematics and make a project showing where three aspects of nature of mathematics - formalism, logic, intuition is applied in the development of mathematics.

Scheme of Evaluation

The following weightage are assigned for evaluation at Higher Secondary Stage in mathematics:

Theory Examination : 80 marks
Internal Assessment : 20 marks

1. Internal assessment of 20 marks, based on school based examination will have following break-up:

Year-end assessment of activities : 12 marks
Assessment of Project Work : 5 marks
Viva-voice : 3 marks

- **Assessment of Activity Work**

(a) Every student will be asked to perform two given activities during the allotted time.

(b) The assessment may be carried out by a team of two mathematics teachers, including the teacher who is taking practical classes.

(c) The break-up of 12 marks for assessment for a single activity may be as under:

- Statement of objective of the activity : 1 mark
- Material required : 1 mark
- Preparation for the activity : 3 marks
- Conduct of the activity : 3 marks
- Observation and analysis : 3 marks
- Results and Conclusion : 1 mark

Total : 12 marks

(d) The marks for two activities may be added first and then marks calculated out of 12.

(e) Full record of activities may be kept by each student.

- **Evaluation of Project Work**

(a) Every student will be asked to do at least one project based on the concepts learnt in the classroom.

(b) The project may be carried out individually (or in a group of two or three students).

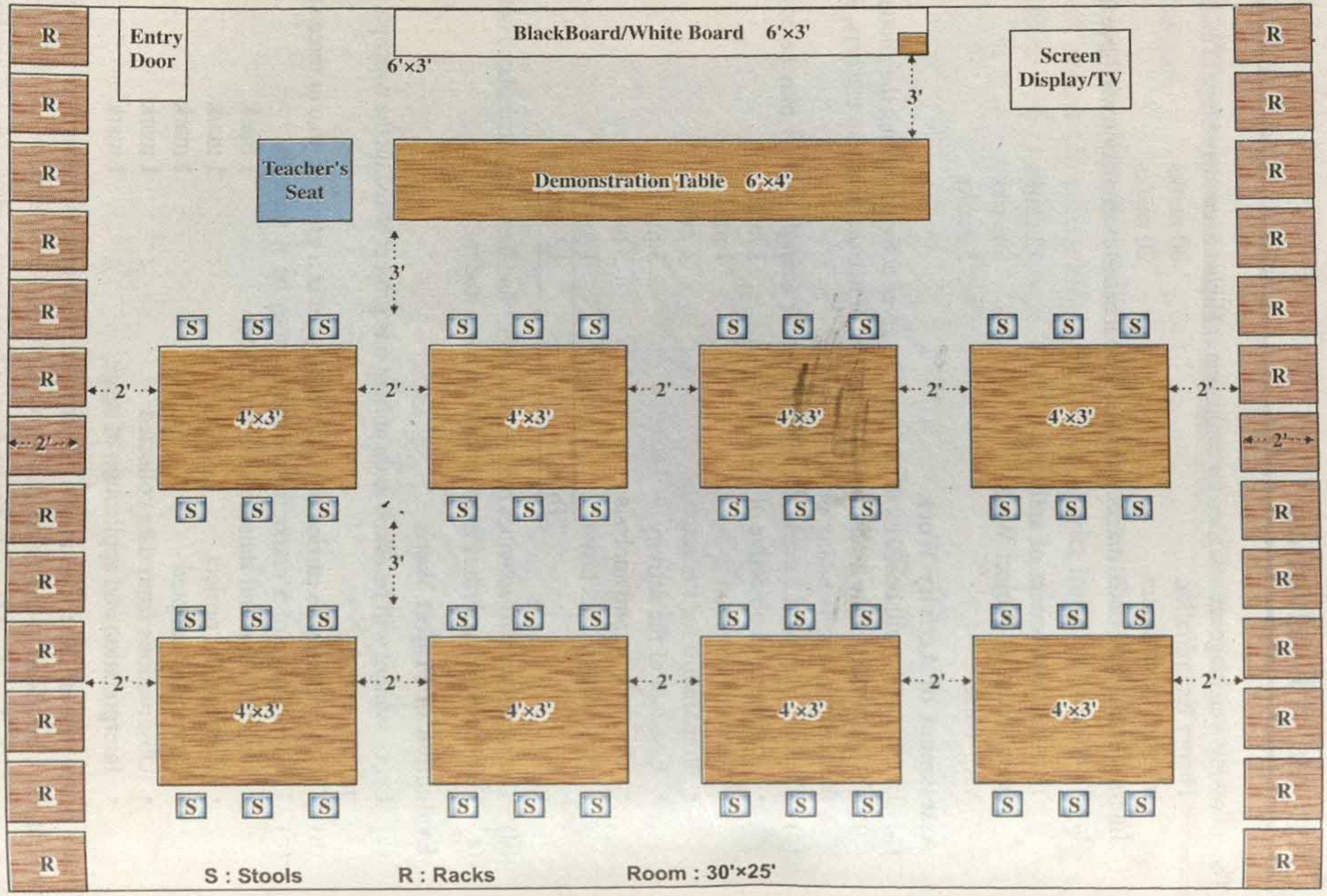
(c) The weightage of 5 marks for the project may be as under :

- Identification and statement of the project : 1 mark
- Planning the project : 1 mark
- Procedure adopted : 1 mark
- Observations from data collected : 1 mark
- Interpretation and application of result : 1 mark

Total Score out of 20 : The marks obtained in year-end assessment of activities and project work be added to the marks in viva-voice to get the total score out of 20.

Note : Every student should be asked to perform at least twenty activities in one academic year.

Set-up of Mathematics Laboratory



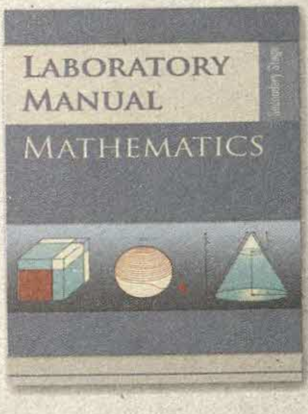
Other Laboratory Manuals by NCERT

In English

- Science Laboratory Manual - Class IX and X
- Chemistry Laboratory Manual - Class XI and XII
- Mathematics Laboratory Manual (Secondary Stage)
- Physics Laboratory Manual - Class XI
- Biology Laboratory Manual - Class XI

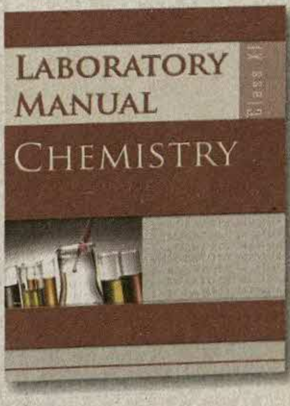
In Hindi

- *Vigyan Prayogshala Pustika* - Class 9 and 10
- *Rasayan Prayogshala Pustika* - Class 11 and 12
- *Ganit Prayogshala Pustika (Madhyamik Star)*



LABORATORY MANUAL
MATHEMATICS

प्रयोगशाला पुस्तिका
रसायन
कक्षा 12



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